# Mathematics & Computer Science

THE COLLEGE OF
WOOSTER

**ANNUAL REPORT** **2014-2015**


*Celebrating I.S. Monday*


*AMRE teams visit the test track at Goodyear Tire & Rubber Co.*

*CS students working in the new 35-seat laptop enabled collaborative lab*

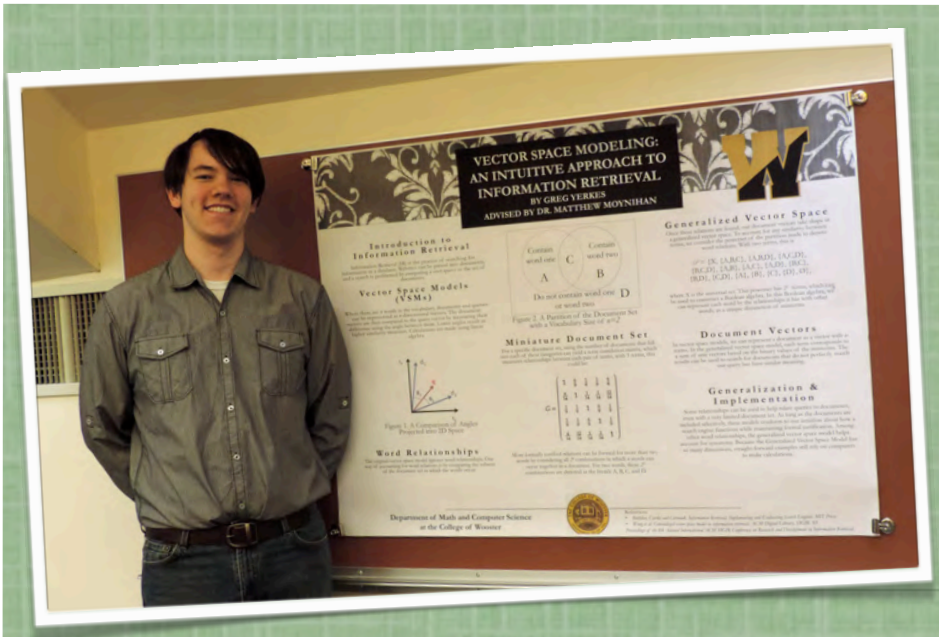

*Math Club T-shirt*

*3D silliness*

# Senior Independent Study

## Vector Space Modeling: An Intuitive Approach to Information Retrieval

**by Gregory Yerkes**
**Advised by Matt Moynihan, Math**

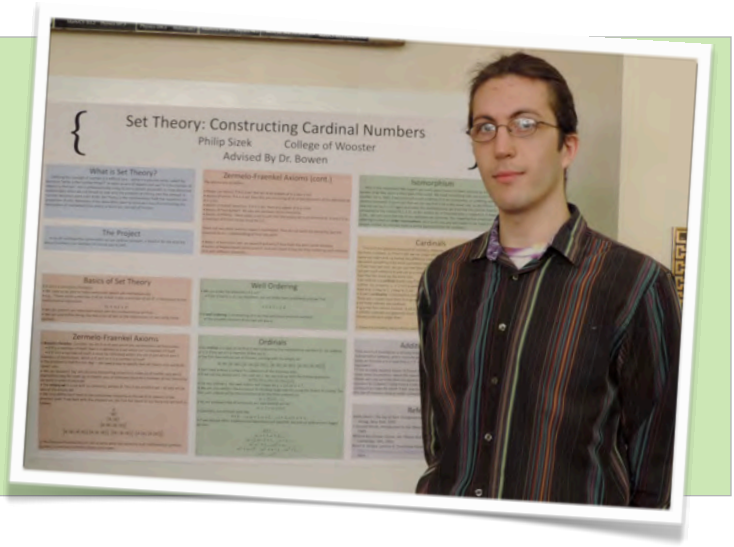



The purpose of this independent study is to introduce readers with a strong enough background in mathematics to the field of information retrieval. A variety of perspectives are introduced that explore the motivation and background of information retrieval. The focus becomes devising a model using some techniques from linear algebra, then reasons for other models are introduced. In the final chapter, more advanced techniques derived from discrete mathematics are employed to generalize this model and make a model that is theoretically consistent and still intuitively appealing. The literature referenced serves a variety of purposes, providing general overviews of the models and as more specific instances of their development. By tracing a development of one of the simpler models, the reader should find the study of IR more accessible.

## Set Theory Building: Numbers from Nothing

**by Philip Sizek**
**Advised by Jennifer Bowen, Math**

Set theory is the field of study surrounding sets, and in this particular development, the study of sets as they relate to the foundation of mathematics and the construction from sets of a workable substitute for the concept of number. This work builds a theory of cardinal numbers using nothing but basic logical operations along with the primitive notions of set and membership. This is accomplished in four parts: the development of the Zermelo-Fraenkel Axioms of set theory; the introduction of fundamental concepts regarding sets and the construction of the ordinal numbers using the axioms introduced; the development of three additional fundamental concepts in set theory; and the description of the concept of cardinal numbers as a construction analogous to natural numbers along with the development of basic arithmetical operations on both the ordinal and cardinal numbers.

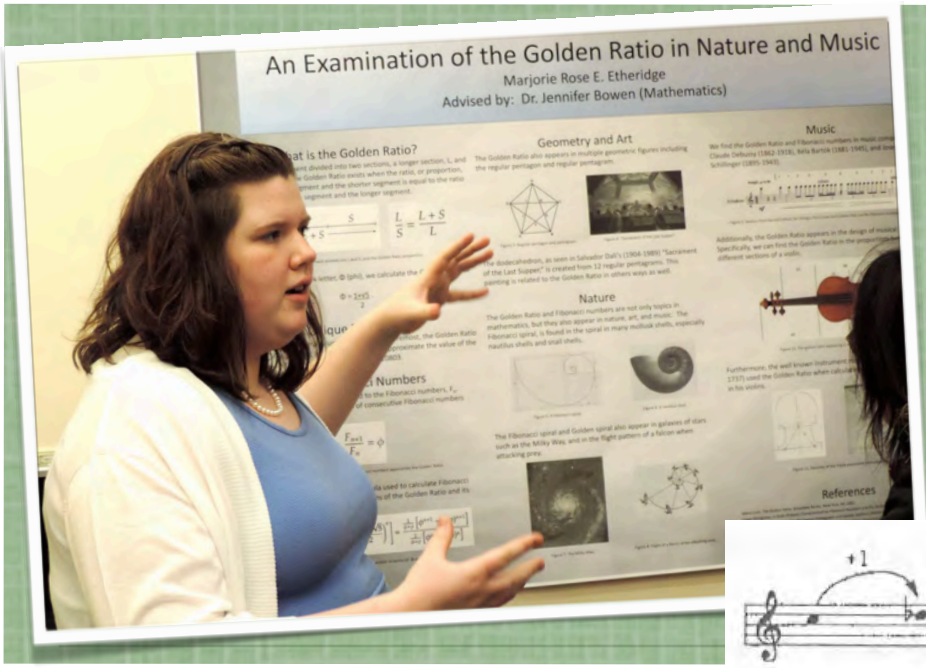


# Senior Independent Study

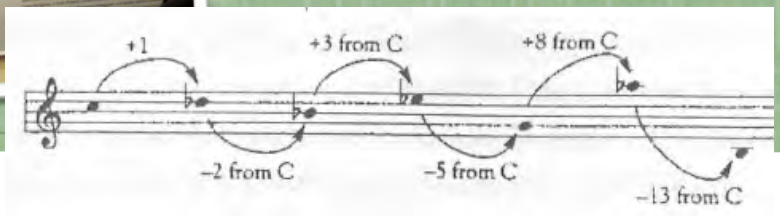
## An Examination of the Golden Ratio in Nature and Music

**by Marjorie Etheridge**

**Advised by Jennifer Bowen, Math**




The Golden Ratio, also known as the Golden Section, exists as a proportion of lengths. Calculated to be approximately 1.61803, the Golden Ratio is closely related to the Fibonacci numbers and other topics in mathematics such as the continued fraction. In addition to its relationship to the Fibonacci numbers, the Golden Ratio is closely connected to many geometric shapes including the regular pentagon, the regular pentagram, certain rectangles, and the dodecahedron. Often thought to have an aesthetically pleasing effect, the Golden Ratio can be found in nature, paintings, music composition, and musical instruments.

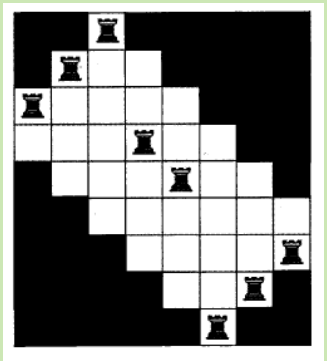


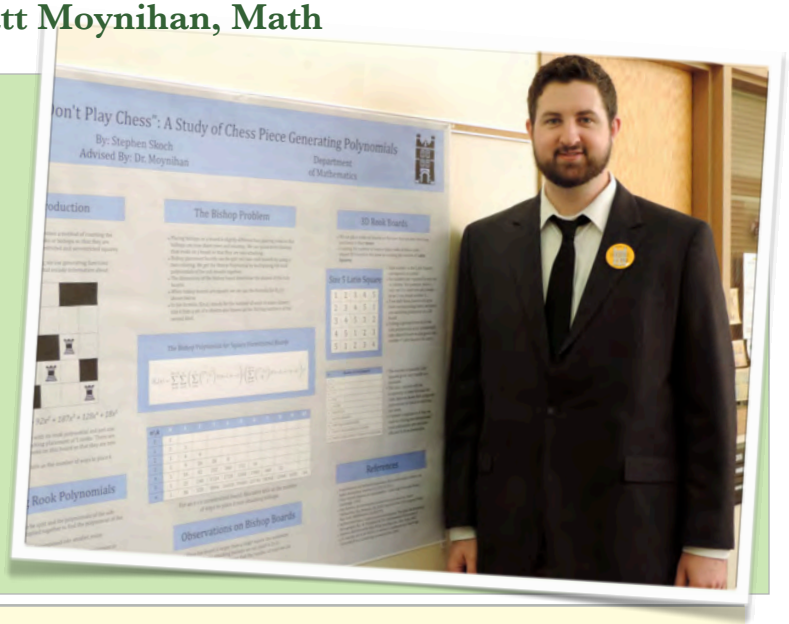

## I Don't Play Chess: A Study of Chess Piece Generating Polynomials

**by Stephen Skoch**

**Advised by Matt Moynihan, Math**

This independent study examines counting problems of non-attacking rook, and non-attacking bishop placements. We examine boards for rook and bishop placement with restricted positions and varied dimensions. In this investigation, we discuss the general formula of a generating function for unrestricted, square bishop boards that relies on the Stirling numbers of the second kind. We discuss the maximum number of bishops we can place on a rectangular board, as well as a brief investigation of non-attacking rook placements on three-dimensional boards, drawing a connection to Latin squares.





# Senior Independent Study

## The Scope of Fractals: Geometry, Iteration, and Application

**by Sebastian Weber**

**Advised by Jennifer Bowen, Math**


Fractals are everywhere. Fractals relate to many branches of science and mathematics. They related to topology and geometry due to their displaying similarity on multiple scales. The dimension of a fractal is a point of discussion, as the creation of a fractal involves a theoretically infinite iteration. We can find many fractal shapes within the complex plane. The most famous of these are the Julia and Mandelbrot set. There is potential for fractals to be used in the world, assisting in understanding phenomenon and updating technology. Differing forms of fractals are seeing use in engineering, economics, and ecology.

## The Cost of a Child: A Bayesian Model of the Fertility Decisions of Women in Academia

**by Christine Hagan**

**Advised by Robert Wooster, Math**

As women have become an increasing percentage of the labor force, many studies have been conducted to examine the effects of women's labor force participation on fertility rates. The decisions each woman makes regarding children is said to be, in part, based on the opportunity costs associated with having a child. Each woman experiences unique opportunity costs which are continuously changing throughout her career. Using a previously created model and data collected from female faculty members at various institutions throughout the United States, this study will explore the fertility decisions made by women in academia.

## Why Apply: Building a Predictive Model to Quantify the Factors that Lead a Domestic Student to Apply to The College of Wooster

**by Joshua Foerst**

**Advised by Drew Pasteur, Math**

This study focuses on the building of a predictive model based on data provided by The College of Wooster Office of Admissions for the recruiting Class of 2018. Model comparison and variable analysis techniques are discussed and employed to build the most accurate logistic model given the data to which the study had access. Literature on the topic is reviewed and discussed in three parts. Student factors review what characteristics a student may have that lead them to be more or less probably to apply to college in general. Institutional factors are those that pertain specifically to The College of Wooster. Interactive factors deal with the interactions between the institution and the student. This literature is used to develop an understanding of the potential significance of variables. Defense is given for the use of a logistic model and the techniques used to build it. Application techniques are also discussed in the actual computation necessary to organize the provided data into categorical variables and then use those variables to build the model. Suggestions are then made to continue this study and improve on its results.

# Senior Independent Study

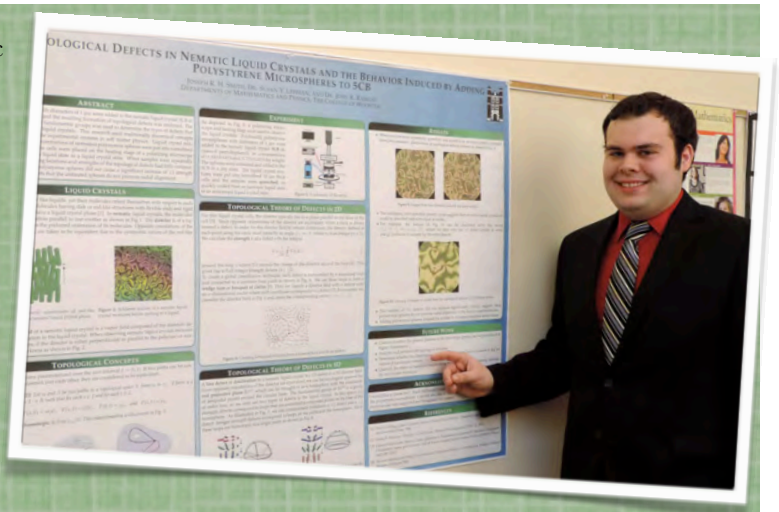
## Topological Defects in Nematic Liquid Crystals and the Behavior Induced by Adding Polystyrene Microspheres to 5CB

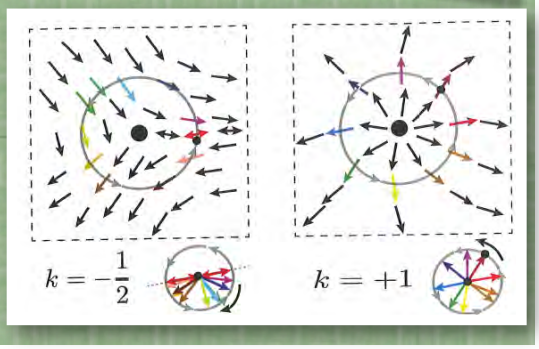
**by Joseph Smith**

**Advised by John Ramsay, Math, and Susan Lehman, Physics**

Polystyrene spheres with diameters of 1 μm were added to the nematic liquid crystal 5CB (4-cyano-4′-pentylbiphenyl) in varied concentrations and the resulting formation of topological defects was analyzed. The topological concept of fundamental groups was used to determine the types of defects that are stable in nematic liquid crystals. This research used traditionally theoretical concepts in topology to describe experimental systems in soft matter physics. Liquid crystal mixtures with varying concentrations of untreated polystyrene spheres were put into nonrubbed 10 μm thick cells. The cells were placed on the heating stage of a polarizing microscope and quenched from a liquid state to a liquid crystal state.

When samples were repeatedly quenched from 40°C to 30°C, we found that the resulting locations and strengths of the topological defects had little variance. This suggests that the inner surfaces of the cells treated with indium tin oxide and polyimide PI 2555 play a major role in the alignment of the molecules. The novel technique of using bouquets of circles to describe the topological defects in a Schlieren texture was applied to regions of our samples. The consistency over repeated quench cycles suggests that using bouquets of circles is a promising method for describing these textures. The addition of the polystyrene spheres did not cause a significant increase of ±1 strength defects, which suggests that the untreated spheres do not promote radial alignment. We also observed that the addition of the spheres lowered the transition temperature between the nematic liquid crystal phase and the isotropic liquid phase, although additional work is needed to isolate the cause and quantify this effect. This thesis served as a new exploration into the area of topological defects both experimentally and theoretically and it has provided multiple directions for future study.






## The Mental Game: An Analysis of Game Theory and Baseball

**by Ian Vernier**

**Advised by Robert Wooster, Math**


Game Theory is a field of mathematics concerned with the analysis of strategies for dealing with competitive situations. Game Theory can be applied to any situation and contains many theorems and properties, such as Nash Equilibrium and the Minimax Theorem, which are used to solve games in matrix form. In addition, the goal of Game Theory is to teach one how to make the best decision possible given certain information. Game Theory lends itself to be applicable for any environment and can be used in a multitude of ways. I use Game Theory to explore baseball situations, specifically with the relationship between a hitter and a pitcher. In this study, I examine specific pitch counts and how each player should approach those counts using the ideas and principles of Game Theory.

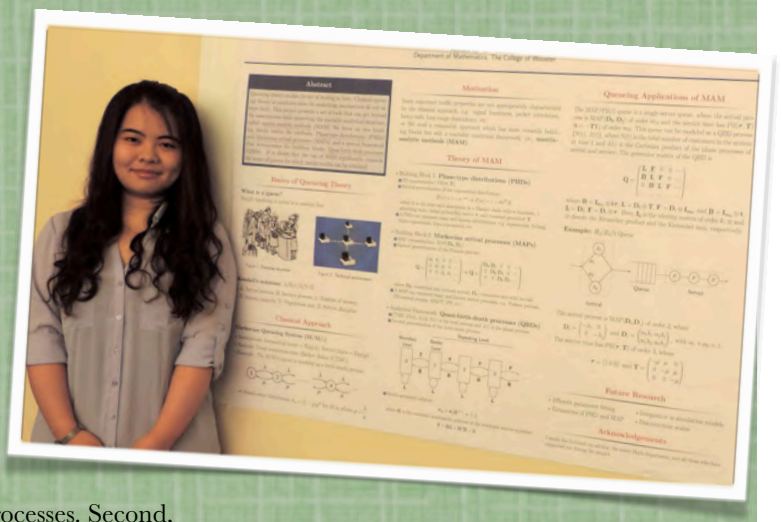
# Senior Independent Study

## Matrix-Analytic Methods in Queueing Theory

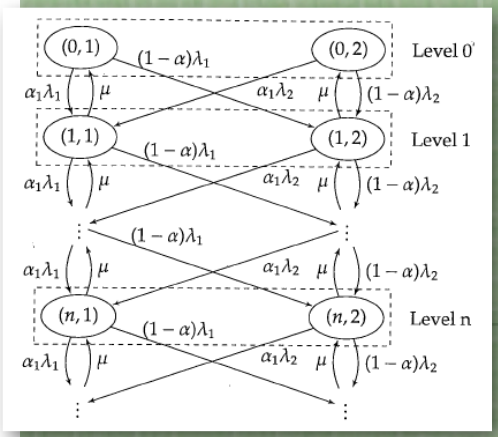
**by Anqi Huang**

**Advised by Robert Wooster, Math**

Analytical methods for tractable (Markov) queueing models commonly assume Poisson arrivals and exponential service. The assumptions are unrealistic from a modeling point of view. In this thesis, we present a set of tools that can get beyond the two assumptions while preserving the underlying Markovian structure, called *matrix-analytic methods.*



Our approach is three-pronged. First, we propose two *building blocks* within the methods, phase-type distributions (PHDs) and Markovian arrival processes (MAPs), which are natural generalizations of the exponential distribution and the Poisson process. They are flexible models that fit nicely into Markov processes. Second,

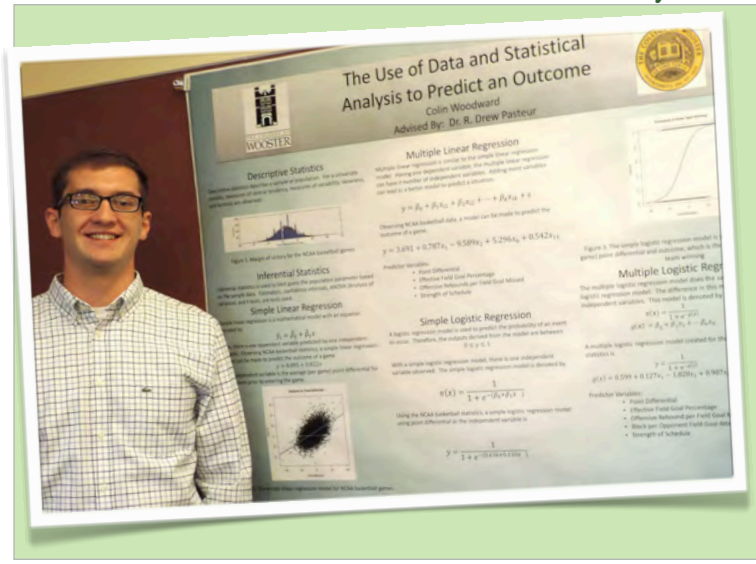



we propose a special *framework* within the methods, quasi-birth-death process (QBD), which is a matrix-generalization of the birth-death process (BDP). Exploiting the structure in the matrix can reduce computational complexity and provide algorithmic tractability. Finally, we demonstrate the applicability of the methods by examining several applied queueing models. We show that the use of PHDs and MAPs in system representation and the QBD in their analysis significantly expand the scope of queueing systems for which usable results can be obtained. The MAP/PH/1 queue is emphasized with a numerical example presented.

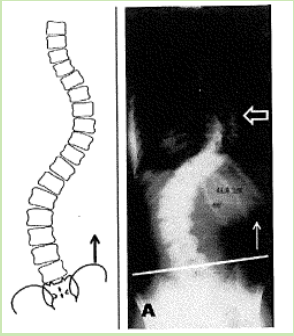
## The Use of Data and Statistical Analysis to Predict an Outcome

**by Colin Woodward**

**Advised by Drew Pasteur, Math**




The ability to make better predictions with the use of data and statistical analysis is used in a variety of fields. For instance, predictions can be made on the outcome of surgery in adolescent idiopathic scoliosis patients with the use of the Scoliosis Research Society (SRS) questionnaires. In sports, the capability of predicting the outcome of a game can be produced. Through the use of regression, models can be created to predict outcomes. More specifically, linear and logistic regression models are used to predict outcomes. An in-depth analysis on various statistical tests and mathematical models will be examined and applied to a specific data set.

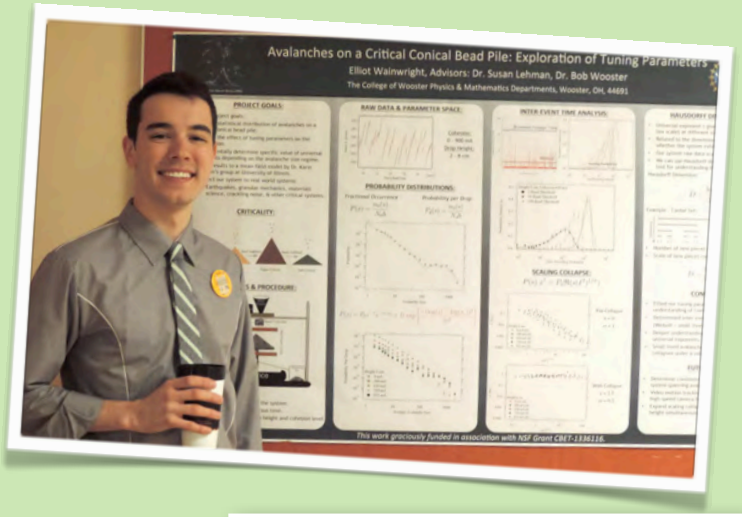


# Senior Independent Study

## Avalanches on a Critical Conical Bead Pile:
## Exploration of Tuning Parameter Space and Mathematical Foundations

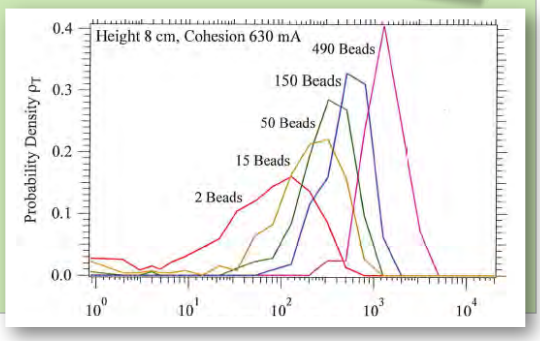
**by Elliot Wainwright**

**Advised by Robert Wooster, Math, and Susan Lehman, Physics**




The effect of tuning parameters on a critical conical bead pile, specifically the effect of drop height and inter-particle cohesion on the avalanche size distribution, has been explored. The pile is slow driven by dropping one steel bead on the apex of the pile at a time. The cohesion of the pile is generated using variable current through Helmholtz coils surrounding the pile. We have compiled data at drop heights between 2 cm and 8 cm at 2 cm intervals, and current values of 0, 500, 630, 750, 835, and 900 mA. We observe the interplay of these two tuning parameters across our entire parameter space, with areas of particular interest at the maximum and minimum drop heights of 2 cm and 8 cm respectively.

The avalanche distributions observed are consistent with previous research, and the analysis has been expanded to calculations of fractional occurrence, probability per drop, complimentary cumulative distribution functions, ratio of total number of avalanches to total bead drops, inter-event time, and angle of repose. When using scaling functions to collapse multiple fractional occurrence runs under cohesion variation, we find that a universal tau exponent value of 1.5 successfully eliminates small avalanche regime power law behavior. For inter-event time avalanche probability densities, the system-spanning avalanche regime is well fit to a Brownian passage-time distributions, while the mixed avalanche regime is well fit to a Weibull distribution.

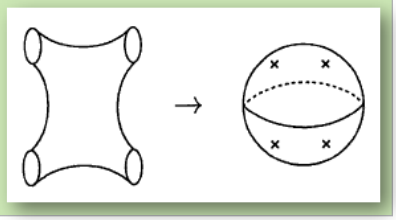



## Modifying the Virasoro-Shapiro Amplitude to Correspond to the Glueball Trajectory

**by Brian Maddock**

**Advised by Jim Hartman, Math, and Nelia Mann, Physics**

High energy proton-proton scattering is dominated by the exchange of glueballs along a leading Regge trajectory. This contribution can be calculated using the particle scattering amplitude. In 26 dimensional bosonic string theory, the corresponding closed string amplitude has similar structural features to the particle amplitude, but corresponds to a non-physical trajectory. In this project we take the closed string amplitude and modify its trajectory to correspond to the glueball trajectory, while maintaining the other important features that it contains. We find a family of closed string amplitudes related by a scaling factor which meets our requirements.



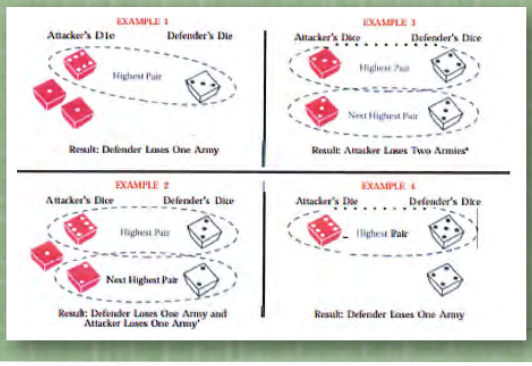
# Senior Independent Study

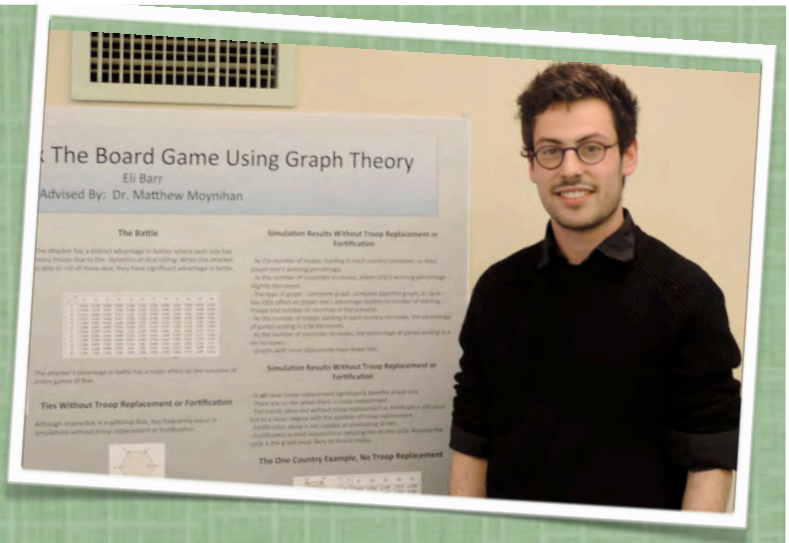
## Read at Your Own Risk:
## An Analysis of Risk the Board Game Using Graph Theory

**by Eli Barr**

**Advised by Matt Moynihan, Math**

The Risk game board can be naturally modeled using graph theory. Each country can be considered a vertex and if two countries are adjacent we can say an edge exists between them. By creating a computer program that can generate new boards and simulate games of Risk on these boards, it can be determined how the shape of the board can benefit or disadvantage a player. Three classes of graphs or boards are examined: the complete graph, the complete bipartite graph, and the cycle graph. In addition to examining the shape of the board, the battle is also analyzed. Exact winning probabilities for the attacker and defender in battle are determined as well as expected troop loss values. The role of the individual battle in the game as a whole is closely examined.




## XH-π bonding in systems of indole and benzene using IBBCEAS and a bioinformatics analysis of protein secondary structure using graph theory

**by Meredith Schervish**

**Advised by Drew Pasteur, Math, and Karl Feierabend, Chemistry**

Analysis of protein structure is one of the most important problems in biochemistry today. A variety of different non-covalent amino acid interactions can lead to complex folding patterns, which in turn lead proteins to function in different ways. One way individual amino acid resides can interact is via XH-π bonding between a hydrogen attached to an electronegative atom on one molecule and a conjugated aromatic in another molecule. This interaction will be investigated through the analysis of indole, a molecule similar to the side chain of tryptophan, and benzene using Incoherent Broadband Cavity-Enhanced Absorption Spectroscopy (IBBCEAS). By analyzing the N-H stretch of indole in the near-IR region, the amount of free N-H and the amount of H-bonded N-H can be determined and used to calculate the equilibrium constant for the interaction. In addition, computational techniques can be used to predict interactions and folding patterns. In order to get comprehensive structural data for a protein, bioinformatic techniques using graph theory will be implemented. Graph spectral theory is the use of a matrix representation of a graph to determine properties of the graph and it will be used to determine clustering within a graph. Cliques are complete subgraphs (or graphs where every vertex is adjacent to every other vertex within the graph). Using graph spectral theory and cliques, proteins with an unknown structure will be compared to proteins with a known structure to determine common features and speculate on function.

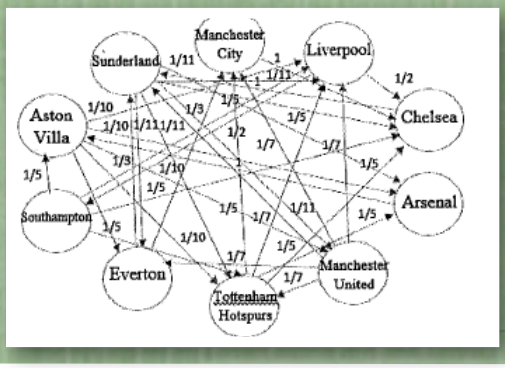
# Senior Independent Study

## Glory, Glory Fan United! A Study of the Relationship Between Fan Loyalty and Team Performance in English Premier League Football

**by Priyanka Datta**

**Advised by Jim Hartman, Math, and Amyaz Moledina, Economics**

English football (soccer) is a unique industry in which the customers or fans are more than just consumers of the sport. While previous studies have focused on the effect of team performance on fan loyalty, there exists evidence to show that fan loyalty, in the form of audience support, motivates the team to perform better on the field and thus, the league. This study analyzes the relationship between fan loyalty and team performance using various statistical methods including Ordinary Least Squares, Two Stage Least Squares and Granger Causality. Mathematical ranking methods developed by Massey and Markov are used to evaluate a team's performance in each season.




## Modeling Crime Using Social Interaction

**by Daniel Miller**

**Advised by Jim Hartman, Math, and James Burnell, Economics**

Crime as a phenomena exhibits spatial variance that is both large and persistent. This paper explains that variance by constructing a model of criminal behavior in which group or neighborhood behavior exists in one of multiple equilibriums. This binomial discrete choice model incorporates the concept of social interactions into previous models of crime by including the effect of criminal activity in a neighborhood on a resident's criminal behavior. Analysis of social interactions between individuals defines an aggregate interaction to simplify a multi-body problem into a single-body problem. Equilibrium is determined by private incentives, the degree of social interactions, and the unobserved heterogeneity among the actors. Equilibrium is derived using Brouwer's fixed point theorem for simplotopes; Brouwer's fixed point theorem is in turn proved through the use of Sperner's Lemma. The degree of social interactions present in Chicago neighborhoods with regards to both violent and property crime is also measured through a logistic regression analysis. Significant social interactions effect was found with regard to motor vehicle theft, theft, burglary, assault, and robbery Rather than viewing social and economic explanations of crime as incompatible, the model integrates other criminological theories into the rational actor framework.

# Senior Independent Study

## To Flip or Not to Flip? A Study of the Effectiveness of the Flipped Classroom Model for Teaching Mathematics

**by Lindy Howard**

**Advised by Matt Moynihan, Math**


Teachers in classrooms worldwide have started to use a newer approach to teach students, known as the flipped classroom model. This model can be described as inverting the classroom such that events which traditionally take place inside the classroom now take place outside the classroom and vice versa. A mixed method study was conducted to examine the effectiveness of the flipped classroom model. This study hoped to accomplish two things: 1) provide an understanding of whether the flipped classroom model affected fourth grade students' learning of mathematics and 2) develop recommendations based on current best practices in mathematics education that teachers may use to guide their future teaching.

## Conquering Carcassonne

**by Andrew Hoover**

**Advised by Drew Pasteur, Math, and Denise Byrnes, Computer Science**


Video games are one of the most popular forms of entertainment in modern-day America, but board games have recently entered a renaissance thanks to the huge success of games like Settlers of Catan, Ticket to Ride, Pandemic, and Carcassone. These two forms of entertainment have intertwined as board game creators quickly realized the huge potential for electronic versions of their games. Due to the fact most board games require multiple players, developers have been forced to include computer-controlled "players" in their games. This project focuses on the development of an electronic version of the modern-day board game, Carcassonne, and an artificial intelligence (AI) that is capable of playing Carcassonne at a competitive level. The rules of Carcassonne are detailed, and a variety of gameplay strategies are discussed. The basics of the mathematical field of game theory are also investigated. The implementation of the game, and it s two bots, "beginner" and "advanced," are also discussed. Finally, the statistics from hundreds of bot vs. bot game simulations, and a moderate amount of human vs. bot games, are analyzed. By the end of this paper, the reader should have a fundamental understanding of the gameplay mechanics and common strategies of Carcassonne, the basics of game theory, and the thought process that went into developing two different bots that can play Carcassonne at a competitive level.

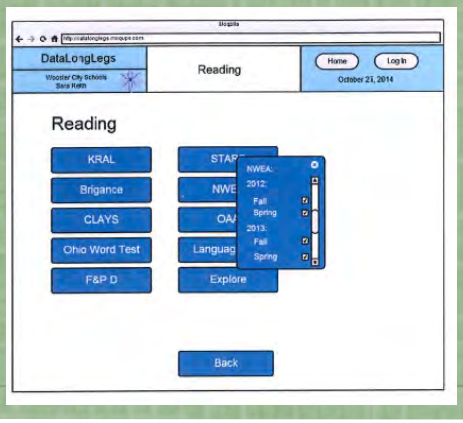
# Senior Independent Study

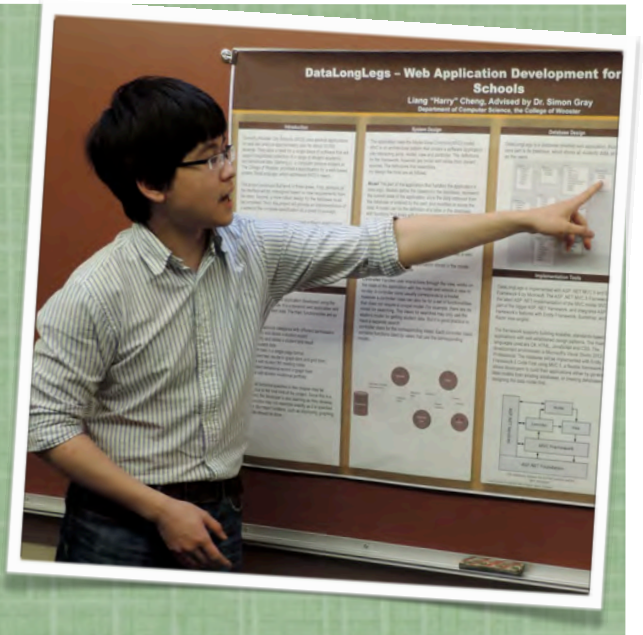
## DataLongLegs - Web Application Development for Wooster City Schools

**by Liang "Harry" Cheng**

**Advised by Simon Gray, Computer Science**

Currently, Wooster City Schools (WCS) uses several applications to track and analyze data for about 10,000 students. They have a need for a single piece of software that will support longitudinal collection of a range of student academic and behavioral data. A previous computer science student at The College of Wooster provided a specification for a web-based system, DataLongLegs, which addresses WCS's needs. This project continues that work in three areas. First, portions of the interface will be redesigned based on new requirements from the client. Second, a more robust design for the database must be completed. Third, this project will provide an implementation of a subset of the complete specification as a proof-of-concept.



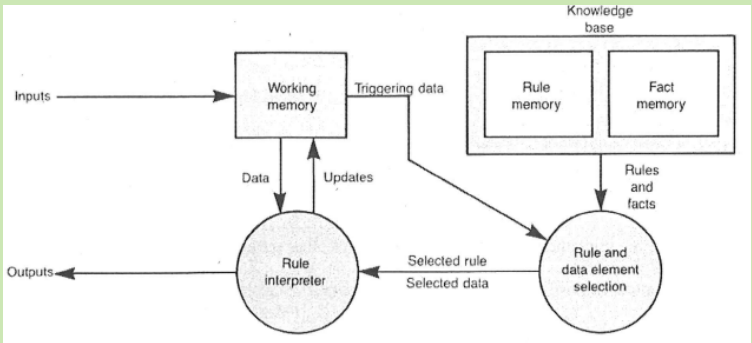




## ScotBoard: An Intelligent Rule Based System for the Exploration of Student Opportunities at The College of Wooster

**by Vernon Zidana**

**Advised by Simon Gray, Computer Science**

Advising students on curricular, co-curricular and extra-curricular activities can be challenging, in large part because it is difficult to locate information about offerings. This is especially true for prospective students, first year students, and new faculty who are not yet familiar with the College's offerings or where to find them. The purpose of this project is to develop an intelligent web application that will 1) allow exploration of curricular, co-curricular and extra-curricular opportunities available at The College of Wooster; and, 2) simplify the construction of a 4-year plan based on a student's personal and academic interest.



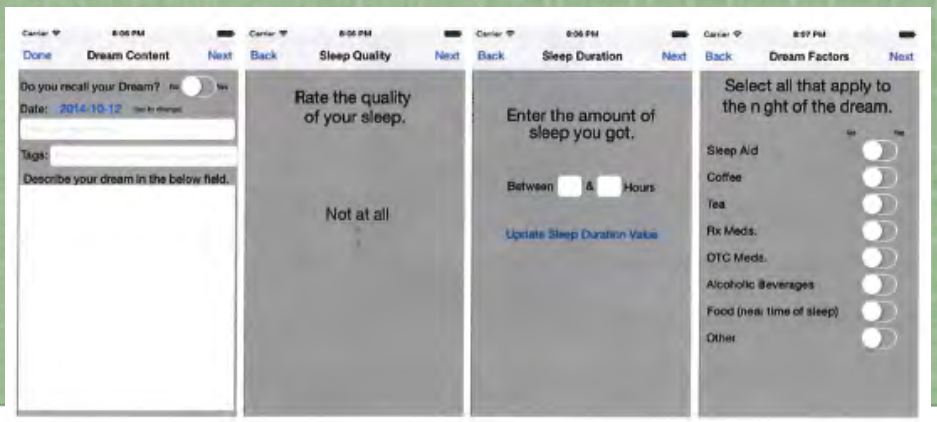
# Senior Independent Study

## iDreamer: Sleep, Dream, Log

**by Clinton Eriksen**

**Advised by Simon Gray, Computer Science**

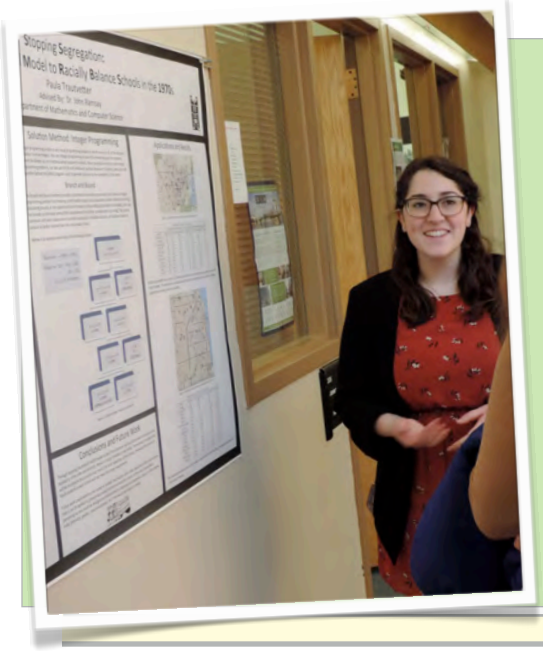
This Independent Study explores the development of an iOS application, from storyboarding to implementing software engineering models. The application created, iDreamer, delves into Oneirology- the study of dreams. Its purpose is to offer a simple way of logging dreams while sophisticatedly analyzing and categorizing them according to the specification of a client. This IS also investigates the life cycles that software proceeds through during the development process, and finally, examines the competitive marketplace of mobile apps.



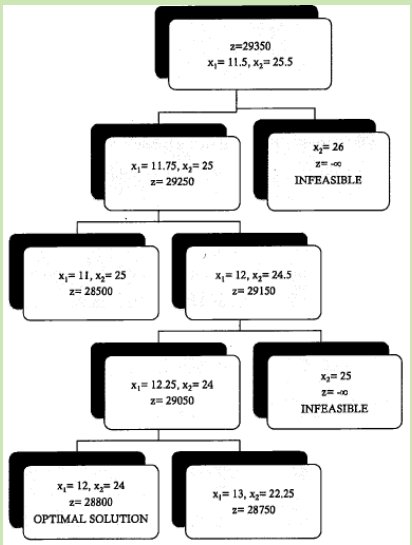

## Stopping Segregation: Using a Mathematical Model to Racially Balance Schools in the 1970s

**by Paula Trautvetter**

**Advised by John Ramsay, Math**



This thesis is an analysis and evaluation of a procedure used in the 1970s to desegregate schools in Gainesville, Florida. This approach to desegregation is described in *A Network-Flow Model for Racially Balancing Schools* by Peter C. Belford and H. Donald Ratliff. This procedure includes a mathematical model for assigning students to schools while balancing race in each school. The model minimizes the number of miles traveled by all students, thus minimizing bussing travel costs. In this thesis, we replicate the Gainesville model using different data and apply the model to Evanston, Illinois to investigate its application in another city.

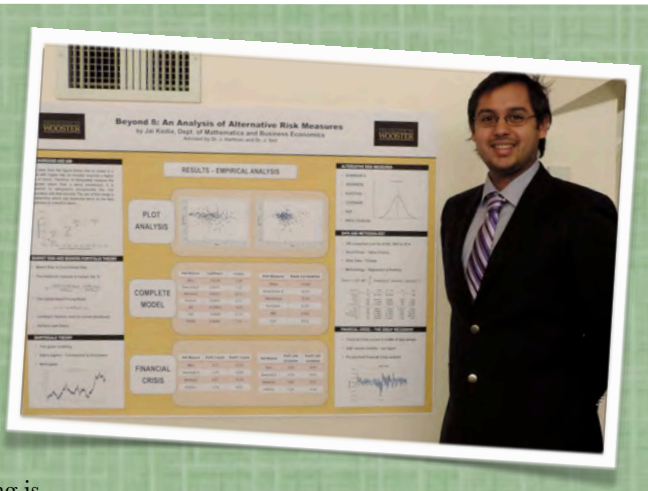


# Senior Independent Study

## Beyond β: An Analysis of Alternative Risk Measures

**by Jai Kedia**

**Advised by Jim Hartman, Math, and John Sell, Business Economics**

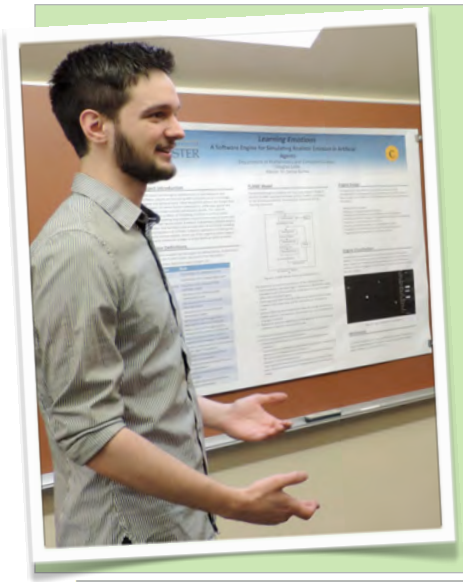



Investors require a return from investing in stock securities that adequately compensate the investors for the risk level assumed. Therefore, any calculation of expected returns from a stock requires knowledge of the risk of the security. While there is no strong consensus on an ideal risk measure, traditionally risk has been conceptualized as volatility and is measured by the of the stock or portfolio. This paper hypothesizes that alternative risk measures such as higher order moments, size, leverage, and price-to-book value add explanatory power to the when predicting stock returns. Empirical analysis is conducted using both regression and portfolio methodologies and data collected on over 300 NYSE companies. The results demonstrate a clear lack of statistical significance of alternative risk measures in explaining returns and show that the relationship between returns and for the time period 2003 – 2014 is negative. Additional testing is conducted by analyzing the impact of the financial crisis on the results and by changing market indices, neither of which significantly change the results obtained. This paper also builds a theoretical framework that may be used to model stock prices using a martingale process. Such a model provides additional testing methods for the normality of returns distributions.

## Learning Emotions: A Software Engine for Simulating Realistic Emotion in Artificial Agents

**by Douglas Code**

**Advised by Denise Byrnes, Computer Science**



This paper outlines a software framework for the simulation of dynamic emotions in simulated agents. This framework acts as a domain-independent, black-box solution for giving actors in games or simulations realistic emotional reactions to events. The emotion management engine provided by the framework uses a modified Fuzzy Logic Adaptive Model of Emotions (FLAME) model, which lets it manage both appraisal of events in relation to an individual's emotional state, and learning mechanisms through which an individual's emotional responses to a particular event or object can change over time. In addition to the FLAME model, the engine draws on the design of the GAMYGDALA emotional engine for games. Evaluations of the model's behavior over a set of test cases are performed, with a discussion of the model's efficacy in different situations.

# Math Faculty

Drew Pasteur, Jim Hartman, Bob Wooster,Matt Moynihan  Jen Bowen, Mary Jo Kreuzman, Ronda Kirsch, John Ramsay

## Jennifer Bowen

**Associate Professor of Mathematics & Chairperson**

Courses taught: Multivariate Calculus and Abstract Algebra, advisor for 3 Senior Independent Study projects

Dr. Bowen served on the advisory board for the STEM Zone initiative for math, chemistry, and biology students.. She was nominated and appointed to the MAA's Problem Books Series Editorial board, a position she will hold through 2018. Along with current and former knot theory student researchers, Professor Bowen and Professor John Ramsay published:

> Michael Bush, Danielle Shepherd, Joseph Smith, Sarah Smith-Polderman, Jennifer Bowen and John Ramsay, "Braid computations for the crossing number of Klein links" *Involve: A Journal in Mathematics*, Vol. 8, No. 1, 169-179 (2015).

## Jim Hartman

**Professor of Mathematics**

Courses taught: Linear Algebra (2 sections), Calculus II (2 sections), Probability and Statics I, advisor for 4 Senior Independent Study projects

Dr. Hartman continues his heavy involvement in the Advanced Placement Program in calculus by presenting at numerous one-day workshops, teaching AP Summer Institutes, and writing AP Calculus problems. In February of 2015 he did a two-day AP workshop in Guam. In the summer of 2015 he served as the International Exam Leader for the AP Calculus Reading.

Dr. Hartman gave a talk at Kenyon College entitled "Power of Magic Matrices". He was also interviewed by the *Wooster Daily Record* for Pi Day 2015.

## Ronda Kirsch

**Instructor of Mathematics and Math Center Coordinator**

Courses taught: Calculus I, Calculus with Algebra II (2 sections)

Ms. Kirsch served on the advisory board for the STEM Zone inititiative, taught the math component of the Youngstown Early Intervention program, and for the first time was an advisor for two AMRE projects: Progresive Insurance and Western Reserve Group.

# Math Faculty

## Robert Wooster

**Assistant Professor of Mathematics**

Courses taught: Calculus II (2 sections), Functions of a Complex Variable, Real Analysis I, , Transition to Advanced Mathematics, Introduction to Differential Forms for Physics (tutorial), advisor for 4 Senior Independent Study projects

Dr. Wooster gave a talk at Ohio Wesleyan University entitled "Introduction to Pricing Financial Options" as part of the Ohio Speakers Circuit. He served on Campus Council and advised and organized (with Matt Moynihan) three student teams' participation in the Mathematical Contest in Modeling (MCM) competition.

Congratulations to Dr. Wooster who had the highest score of **179** for Math/CS in Taylor Box XXVI.

Math/CS 104.7 Physics 93.0

## Pamela Pierce

**Professor of Mathematics**

On leave 2014-2015

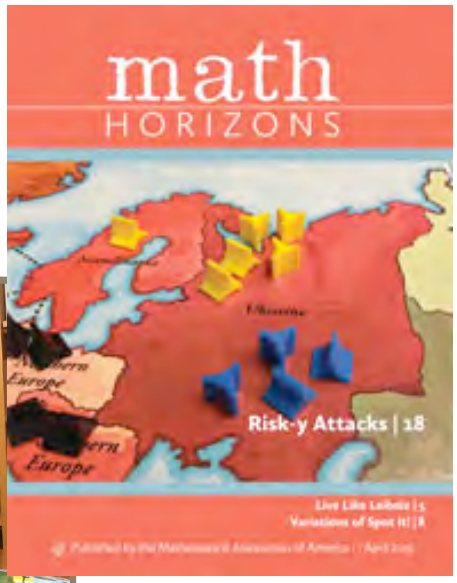
The highlight of Dr. Pierce's leave was a trip to Poland and Slovakia in the fall of 2014. In Slovakia she attended a conference on Real Functions Theory.

She gave a colloquium talk at Hamilton College in the fall and was the invited speaker at the Pi Mu Epsilon Induction Ceremony at Manhattan College in April of 2015.

Dr. Pierce co-authored a book chapter with Simon Gray (Computer Science) and two others:

> Developing Research Skills Across the Undergraduate Curriculum (book chapter, with L. Coates, A. Fraser, and S. Gray) in *Enhancing and Expanding Undergraduate Research: A Systems Approach*, pp. 145-168, 2015.

Dr. Pierce and Dr. Wooster co-authored the cover article of the April 2015 issue of *Math Horizons* entitled "Conquer the World with Markov Chains". This photo of math major Amanda Steinhebel '15 playing the board game Risk was included in the article.




## Mary Jo Kreuzman

**Visiting Assistant Professor of Mathematics**

Courses taught: Transition to Advanced Mathematics, Multivariate Calculus (2 sections)

This past summer, Dr. Kreuzman took part in a trip to Ghana to facilitate the establishment of a college partnership with Ashesi University.

## Matt Moynihan

**Visiting Assistant Professor of Mathematics**

Courses taught: First Year Seminar, Calculus I (3 sections), Transition to Advanced Mathematics, advisor for 4 Senior Independent Study projects

Dr. Moynihan's First Year Seminar was entitled "Calculating Morality" and examined how mathematics plays a frequently overlooked role in our day-to-day lives. While often viewed as pure, cold, and amoral, mathematics bleeds into unexpected disciplines and supports/contradicts various social policies. Rather than focusing on the mathematics itself (which is sometimes difficult and/or potentially classified!), this seminar focused on ethical dilemmas surrounding quantitative arguments that we encounter every day.

Dr. Moynihan gave a talk at the national MAA/AMA conference in San Antonio in January entitled "On Permutation Statistics and Algebras". Also, he had an article accepted by the *Journal of Algebraic Combinatorics* entitled "The colored Eulerian descent algebra". It is available online while awaiting publication in the print version.
http://link.springer.com/article/10.1007/s10801-015-0596-z

# Math Faculty

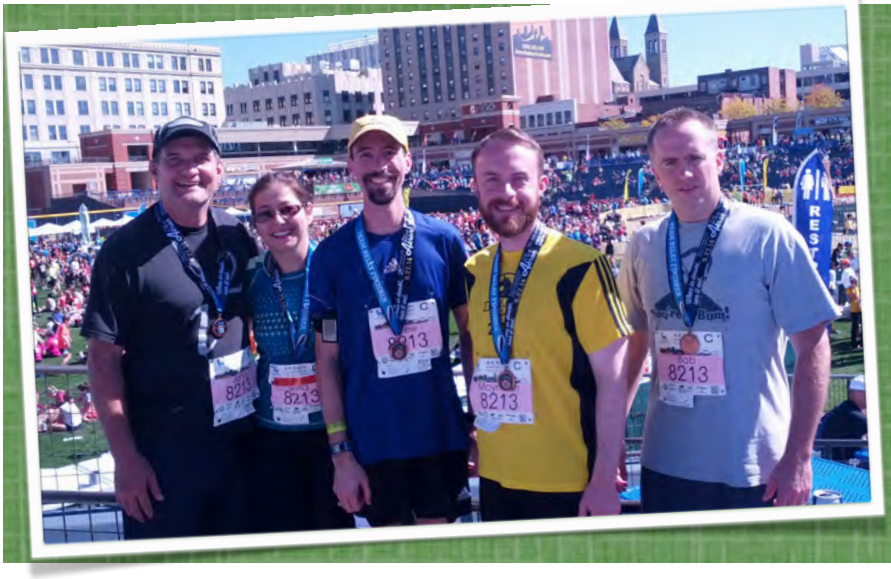
## Drew Pasteur

**Associate Professor of Mathematics and Assistant Director of AMRE**

Courses taught: Calculus with Algebra A (2 sections), Math in Contemporary Society, Math Modeling, advisor for 4 Senior Independent Study projects

Drew Pasteur co-organized (with Bob Wooster) the 2014 Midstates Conference for Undergraduate Research in Computer Science and Mathematics (MCURCSM), held at Wooster last November. He also served as assistant director of the College's summer Applied Mathematics Research Experience (AMRE) program, and as chair of Committee on Committees. Dr. Pasteur served as a reviewer for Journal of Quantitative Analysis in Sports and was co-organizer for "Math and Sports" session at the national Joint Math Meetings.

In Dr. Pasteur's Math Modeling course, he added a heavy emphasis on scientific writing and made substantial revisions to three lab assignments, including using the game-like online data modeling site Kaggle for the first time.



*Wooster Math/CS faculty at 2014 Akron Marathon Relay*

*John Ramsay, Sofia Visa, Drew Pasteur, Matt Moynihan, Bob Wooster*

## John Ramsay

**Professor of Mathematics, Director of AMRE and Dean of Experiential Learning**

Courses taught: Operations Research, Topology, advisor for 2 Senior Independent Study projects

The Applied Mathematics & Research Experience (AMRE) continues to expand under Dr. Ramsay's leadership. Ten projects were completed in the summer of 2015, including two for Goodyear Tire & Rubber Company and a linguistics project for Prentke Romich Co. The program welcomed several new clients this year, including Artiflex Manufacturing and Western Research Group. The program was fortunate to welcome four students from Africa as part of the establishment of a college partnership with Ashesi University in Ghana. Each of these students became a member of a different AMRE team and worked closely with Wooster students and faculty advisors.

*The AMRE 2015 teams learned the importance of "first impressions" at one of several personal/professional development workshops held throughout the summer.*

# Computer Science Faculty

Denise Byrnes, Sofia Visa, Simon Gray

## Denise Byrnes

**Associate Professor of Computer Science**

Courses taught: Multimedia Computing, Data Structures and Algorithms (2 sections), User Interface Design, Algorithm Analysis, advisor for 2 Senior Independent Study projects

## Sofia Visa

**Associate Professor of Computer Science**

Courses taught: Scientific Computing (2 sections), Data Structures and Algorithms lab, Operating Systems, Problem Seminar, Machine Intelligence, Introduction to Bioinformatics, advisor for 2 Senior Independent Study projects

Dr. Visa was the the Program Committee chair for the Ohio Celebration of Women in Computing (OCWiC) held in Huron, Ohio in February. In March, Dr. Visa presented a Faculty at Large lecture to the campus community entitled "Thinking Like a Computer Scientist: Fuzzy Logic, Image-based Search, and Big Tomatoes". Dr. Visa co-authored a paper that will appear in summer 2015:

> E.Van der Knaap, J. Clevenger, J. Van Houten, M. Blackwood*, G. Rodriguez, Y. Jikumaru, Y. Kamiya, M. Kusano, K. Saito, and S. Visa, Network analyses reveal shifts in transcript profiles and metabolites that accompany the expression of SUN and an elongated tomato fruit, to appear in *Plant Physiology*. *Wooster student co-author

Dr. Visa is Co-PI in collaboration with E. van der Knaap of The Ohio State University as part of a $4 million NSF grant to study genetic and epigenetic variation in the regulation of tomato fruit quality traits (2015-2019).

## Simon Gray

**Associate Professor of Computer Science**

Courses taught: Imperative Problem Solving (2 sections), Programming Languages, First Year Seminar, Data Structures lab, advisor for 3 Senior Independent Study projects

Dr. Gray's First Year Seminar, entitled "The Art and Science of Design", explored design as a multi-disciplinary problem-solving activity. Through several projects , students looked a the design process from need identification, through specification and the evaluation of alternatives, to the development of a solution.

Dr. Gray will be departing Wooster and re-joining the Great Lakes Colleges Association as the program officer with responsibility for advancing the goals of the Global Liberal Arts Alliance. With member institutions in fifteen countries, the Alliance includes the thirteen schools of the GLCA and fourteen liberal arts schools outside the states. Dr. Gray is excited to work on programs that connect faculty members from Alliance schools to advance education in the tradition of the liberal arts.

career JOURNEYS

# CS Alums Talk About Life After Wooster

## Thursday, February 26, noon - 1 pm
## CoRE, Andrews Library

**Three Computer Science alumni will talk about their career paths since leaving Wooster. This will be followed by an extended question & answer period.**

## Holly Laiveling

Holly graduated from Wooster in 1993 with a major in Computer Science.
Holly's first position after graduation was an IT consulting job with a large national consulting firm where she transformed legacy mainframe systems into desktop applications. Throughout the course of her career, she has experienced IT consulting, boutique software development, and the creation of her own web development company. Currently she is a Senior Software Developer for dunnhumbyUSA, an innovative customer science company that uses predictive analytics to enhance and reward customer loyalty.

## Michael Rulf

Michael graduated from Wooster in 1993 with a double major in Computer Science and special major Theatre Engineering.
Michael is Core Services' Chief Technology Officer with over two decades experience providing strategic direction and management of large technology teams at multiple Fortune 100 companies. His roles allowed continuous hands-on involvement in design, installation, maintenance and extension of distributed software solutions utilizing SOA principles as well as Cloud and SaaS enablement platforms. His work resulted in designation as an Oracle ACE Director in 2008 and receiving OAUG's Innovator of the Year award.

## Amit Tibrewal

Amit graduated from Wooster in 1990 with a major in Computer Science and a minor in Mathematics.
Amit is a 'lifer' at ITC Limited - a US$ 7.0 billion multi-business Indian conglomerate - having joined them after returning to India in 1990. He has over two decades of varied work experience in the IT industry – selling software solutions and products, working with ITC's business users to implement solutions that help transform the business, marketing, branding, sales, account management and operations. As General Manager of IT Services, apart from monitoring and tracking large enterprise projects across ITC's 13 businesses, Amit is part of the central IT leadership team providing IT technology and strategy for the company.

# Contests & Conferences

In February, The College of Wooster had three teams of math and computer science majors (advised by Bob Wooster and Matt Moynihan) compete in a 96-hour, worldwide math modeling competition. In the contest each team can choose to do an MCM (Mathematical Contest in Modeling) or an ICM (Interdisciplinary Contest in Modeling) problem. Details about the competition can be found here: http://www.comap.com/undergraduate/contests/

In the MCM: 7637 teams participated
In the ICML: 2137 teams participated

The teams from Wooster:

1. Khoa Nguyen '17, Avi Vajpeyi '17, and Alishan Premani '17: Our all-First Year team tackled MCM problem A and earned an ***Honorable Mention*** which put them in the top 31% of the 7637 teams.

2. Anqi Huang '15, Alex Iudice '17, and Jacob Priest '16 chose MCM problem B and earned a ***Meritorious*** rank which means they were in the top 9% of the MCM teams!

3. Popi Palchoudhuri '16, Carlos Gonzalez '16, and Sunny Mitra '16 took on ICM problem C and earned an ***Honorable Mention*** which put them in the top 43% of all teams.

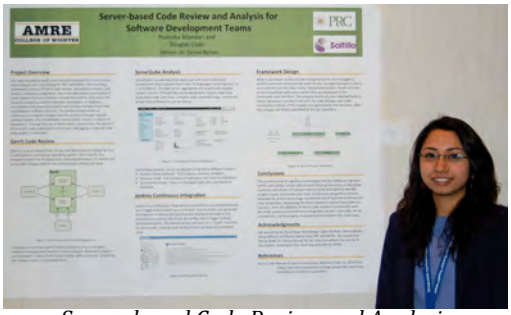
These three teams worked very hard and did an outstanding job representing the College of Wooster on

Computer Science sent two student teams to the ACM Programming Contest in Youngstown, Ohio last fall. The two teams, Team DXC and Team Coding Scots, consisted of Dagmawi Zegeye, Carlos Gonzalez, Emily Sersain, Xiangyu Li, Conor Maley and Doug Code. Congratulations to these students for their participation and hard work!

Our department hosted the annual Midstates Conference for Undergraduate Research in Computer Science and Mathematics (MCURCSM) in November. MCURCSM was well organized by Drew Pasteur, Sofia Visa, and Bob Wooster. Students Brian Foley 17, Carlos Gonzalez '16, and Laith Sersain '16 presented their 2014 summer research as part of the program:

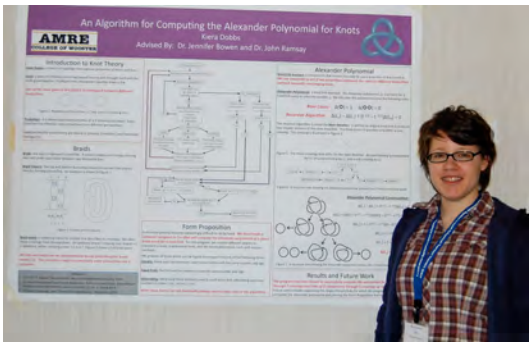
*A Frequency- and Clustering-based Methodology for Finding Transcription Factor Binding Sites*
Carlos Gonzalez, Laith Sersain, and Sofia Visa

*Automated Reidemeister Moves: A Numerical Approach to the Unknotting Problem,* Brian Foley



*Server-based Code Review and Analysis for Software Development Teams*
Pratistha Bhandari

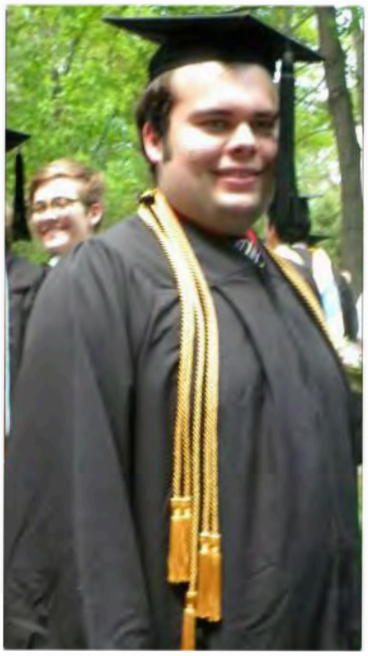
*February 2015, Huron OH*



*Alexander Polynomial Program*
Kiera Dobbs

Kiera Dobbs '16 won Best Poster Award and $1,000 toward her participation at a national event in computing (Grace Hopper meeting organized by Anita Borg Institute) to take place in Houston in October 2015.

# Honors & Awards to Math/CS Majors



## Latin Honors

*Summa cum laude*
Anqi Huang
Joseph Smith
Amanda Steinhebel

*Magna cum laude*
Douglas Code
Andrew Hover
Jai Kedia
Stephen Skoch
Elliot Wainwright

*Cum laude*
Brian Maddock
Daniel Miller
Sebastian Weber

## Phi Beta Kappa

Douglas Code
Anqi Huang
Stephen Skoch
Joseph Smith
Amanda Steinhebel

## Foster Prize in Mathematics

Daniel Miller

## Endowed Faculty Scholarship

Andrew Hoover
Amanda Steinhebel

## William H. Wilson Prize in Mathematics

Joseph Smith and Anqi Huang (above photos)

## Barbara Ward McGraw Memorial Prize

Christine Hagan

## Elizabeth Sidwell Wagner Prize in Mathematics & Computer Science

Stephen Skoch

## Theron Peterson and Dorothy Peterson Award for Outstanding Academic Achievement

Elliot Wainwright

## William A. Galpin Award for General Excellence in College Work

Elliot Wainwright

## Vivien Chan Prize in Interdisciplinary Sciences

Meredith Schervish

## Maria Sexton Award

Lindy Howard

## Andrew Dearborn Cronin Emerging Leader Award

Emily Howerton

## Lyman C. Knight Sr. Prize in Mathematics & Physical Education

Emily Howerton

# Math/CS Colloquium Series

## Math/CS and S-Stem Scholars



WIL RESEARCH

PPPL

NASA

### Summer Research Colloquium

Amanda Steinhebel '15
(Math and Physics)
Plasma physics at General Atomics/Princeton

Libby McInturf '16
(Chemistry)
Pharmaceutical research at WIL Research

Matt King-Smith '16
(Physics)
NASA Internship

- Thursday, September 18
- 4:00 PM
- Taylor 110

Refreshments!!

## Math/CS and S-Stem Scholars



UNIVERSITY OF Cincinnati

NC STATE UNIVERSITY

THE UNIVERSITY OF ARIZONA

### Summer Research Colloquium

Andrew Hoover '15
(Math and Computer Science)
Artificial intelligence in mobile-device games

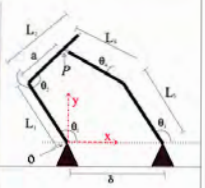
Nick Lesner '16
(Chemistry)
Research in bioanalytical chemistry

Torger Miller '16
(Computer Science)
Autonomous vehicles

- Tuesday, September 23
- 11:00 AM
- Taylor 110

PIZZA lunch following the talks!

## Math/CS Colloquium Series

### *How to solve a polynomial system and why you might care to do so*

Dan Bates
Associate Professor of Mathematics
Colorado State University
College of Wooster '01

Thursday, October 23, 11:00 am
Taylor 111

## Math/CS Colloquium Series

### *Einstein on the Circle*

Christoph Marx
Assistant Professor of Mathematics
Oberlin College

Whether it is a bottle of soda, the tires of a car or the human body.

Objects in everyday life can be described by only a few parameters, like temperature, pressure or volume. But how is this possible, if each of these systems are complex assemblies of atoms and molecules giving rise to a vast number of coordinates, on the order of $10^{23}$?

Ergodic theory is the mathematical attempt to provide an answer to this fundamental question. In this talk we will tackle the problem based on a prominent example - the Einstein model for a solid. It will be shown that this problem is reduced to studying rotations on a circle, for which we will prove a version of the ergodic theorem.

Thursday, November 20, 11:00 am

## Woo Math Alumni Speaker

### *From Mathematical Modeling to Business Strategy: My Career Journey*

Sohil Parekh '99

Sohil Parekh is Senior Director of Strategy and Business Development with Gazelle, based in Boston, MA. He has previously held Strategy roles at companies like Staples and Microsoft and leading management consulting firms BCG and Capgemini. But before all that, he was a Math major at Wooster and a student in Dr. Ramsay's class. In this informal talk, Sohil will reflect on his career path and his attempts at solving "real-world" business problems.

Friday, November 21, 2:00 pm

## Math/CS Colloquium Series

### *Artificial Intelligence of Modern Board Games: Football Strategy*

Sean McCulloch
Associate Professor of Computer Science @ Ohio Wesleyan University

Much of the work that has been done in the area of designing intelligent agents for game playing has been done on "classic" games, such as Chess, Checkers, and Go. Recent years have seen a widespread growth of different kinds of games that require different kinds of techniques. These games often have interesting underlying mathematical structure that can be exploited by a program.

In this talk, I will discuss a game called "Football Strategy", which has an underlying structure that can be analyzed using techniques from Game Theory. I will explain the game theoretic techniques involved, and how they were applied to design a program to play the game. I will also discuss modifications that were made to the program to handle situations that cannot be directly approached using these game theoretic techniques.

Wednesday, February 18, 3:00 pm

## Math/CS Colloquium Series

### *A Hall of Fame Introduction to Logistic Regression*

Rod Sturdivant
Associate Professor of Clinical Public Health
The Ohio State University College of Public Health

The logistic regression model is popular in many fields, particularly in the health sciences, where the outcome is categorical. In this talk, the logistic regression model will be introduced as an approach for predicting election of a baseball player to the Hall of Fame. The features of the model that have made it so useful in other applications are illustrated in a context easily understood by baseball fans and those with no knowledge of the game. Topics discussed include predictive modeling, model building, model assessment, interpretation and validation.

NATIONAL BASEBALL HALL OF FAME

Thursday, March 26, 11:00 am

## Math/CS Colloquium Series

### *Solving Congruences with P-adic Methods*

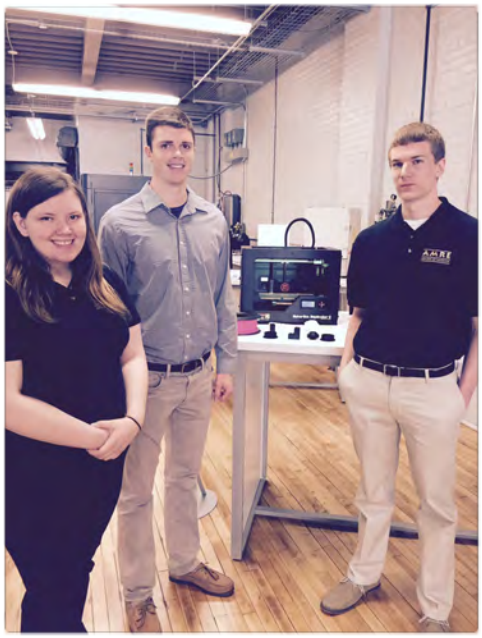
Pamela Richardson
Associate Professor of Mathematics
WESTMINSTER COLLEGE

Many real-world systems employ digital signature schemes, which are processes used to authenticate messages. The security of digital signature schemes often relies on the fact that certain congruences are difficult to solve modulo a prime power, especially when using a large prime. In this talk, we introduce the p-adic integers and explore how they can help us analyze interesting congruences.

Thursday, April 16, 11:00 am

# Applied Mathematics & Research Experience 2015 (AMRE)

Summer of 2015 marked AMRE's 22nd year of successfully giving Wooster students experience in practical applications that the classroom cannot provide. Ten projects were completed by 28 students. The program continues to branch out from the original math, computer science, and economics projects into other areas such as linguistics and marketing.



## Center for Entrepreneurship
## Marketing Entrepreneurship and 3D Printer Market Research

Team members: Marla Walton, Alex Lalonde, Jacob Sparks
Faculty advisors: Peter Abramo (Center for Entrepreneurship) and Kate Gullatta (Learning Center)

The C4E AMRE team assisted the Director of Entrepreneurship in web page design and other marketing aspects of the Entrepreneurship program at The College of Wooster. The team researched entrepreneurship programs at other institutions and researched the feasibility of incorporating potential new initiatives at The College of Wooster. The main initiative that was investigated was a 3D printing program that would train students to act as consultants to business and industrial clients with 3D printing needs. The team developed a plan for the acquisition and implementation of a 3D printer and helped to shape potential program ideas for the 3D printer. The team was able to supply advice on what 3D printers would be of best use for The College of Wooster students. In addition, the team was able to redesign The Center for Entrepreneurship's website to make it more informative and user friendly for both prospective students and current students.

## Progressive Insurance
## Data Analysis for Competitive Intelligence

Team members: Kenneth Mintah, Katarina Kremling, Sophia Anderson, Melissa Griffith
Faculty advisors: John Ramsay and Ronda Kirsch (Math)

The AMRE team had two projects: determining competitiveness in certain markets and analyzing a competitor's tiering model. They were given data from various insurance raters to analyze characteristics of policy holders as well as how well Progressive's premium ranked against other companies. They were also given competitor data to read, summarize, and apply to potential customer profiles. The team detailed their findings in a report to Progressive.

# Applied Mathematics & Research Experience (AMRE)


## Artiflex Manufacturing LLC
## Improving Revenue Forecasting Through Analysis of Historical Data

Team members: Robin Morillo, Ke Song, Pratistha Bhandari, Chloe Acheampong

Faculty advisors: Drew Pasteur (Math) and Harry Michael (Economics)

The AMRE team was hired by ArtiFlex Manufacturing to improve their revenue forecasting model. ArtiFlex manufactures replacement parts for automobiles. Throughout the eight-week long program, the students analyzed production and shipment records of ArtiFlex in comparison to their forecasting model predictions. Using Excel and other computational tools, the team was able to improve many areas of the forecasting model, improving its accuracy in predicting future revenue for ArtiFlex. In addition, the team updated the user interface of the forecasting model by adding features that made viewing and editing the data easier and more efficient.

## Western Reserve Group
## Standard Rate Adjustment Template

Team members: Josh Houtz, Gina Lam, Heather Smith

Faculty advisors: Jen Bowen and Ronda Kirsch (Math)

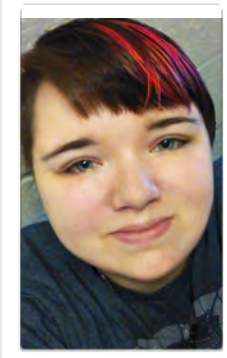
The Western Reserve Group was provided with five Excel files and one Access file including all of the queries, macros, modules, and formulas required to complete the tasks they assigned to the team. These documents can to be used in the future as templates for other applications. The AMRE Team's hope is that the work they have done will be applicable to future projects in other lines of business, including farm and auto insurance.


## Prentke-Romich Company
## Building a Semantic Network for AAC Devices


Team members: Marissa Kobylas and Laith Sersain

Faculty advisor: Diane Uber (Spanish)

The AMRE team was hired to serve as consultants for Prentke Romich Company (PRC), a local company that makes augmentative and alternative communication (AAC) devices for individuals with disorders that prevent or hinder their use of spoken language. PRC is interested in making AAC software and language systems more intelligent by adding an automated recommendation feature that recommends related word sets to ultimately create a more organic learning process. The AMRE team was given the task of researching the applicability of a semantic network, a way to link words together based on their meanings, to AAC devices and developing a methodology for the creation of such a network. After conducting research and collaborating with PRC, the team recommended a hierarchically-structured database that links words together with several different Link types to indicate the type of relation between two given words. The recommendations for the system will allow for both flexibility and growth over time, and for a powerful framework that can adapt to users' needs.



# Applied Mathematics & Research Experience (AMRE)

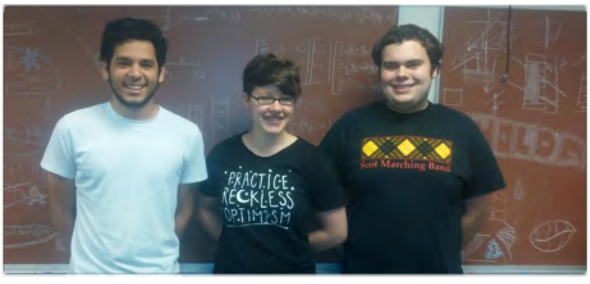

**Goodyear Tire & Rubber Company**
**Automated X-ray Evaluation Process for OTR 57 & 63 Tires**

Team members: Emily Howerton, Alex Iudice, Khoa Nguyen, Obed Kobina Nsiah
Faculty advisors: Drew Pasteur and Matt Moynihan (Math)

The purpose of this project was to provide Goodyear with a software tool to automate X-Ray based quality checks. The software is used to assist operators who observe X-Ray images of large tires for defects that can cause tire failure. Generally performing two major types of analysis (Splice and Waviness) the program generates results that the operators can use to make production decisions. For each type of analysis, a unique filtering method is used to process the X-Ray images into viable formats which make it easier for the computer to perform the analysis. At the end of the project, both analysis were finely integrated into an intuitive user interface to make user interactions with the software easy and interactive.

**Goodyear Tire & Rubber Company**
**Tire Zigzag Belt Winding Optimization and Visualization**

Team members: Carlos Gonzalez, Kiera Dobbs, Joseph Smith
Faculty advisors: Denise Byrnes (Computer Science) and Matt Moynihan (Math)




The purpose of this project was to provide both tools and analysis in regards to optimal aviation tire wrapping. We began with a 2D model of the zigzag belt wrapping and we extended this into three dimensions in order to provide better visualization of the wrapping process. The final task was to create a design space to test possible wrapping patterns. This included using a mathematical approach to reduce the number of possible zigzag patterns to a small, optimal set. The result of this project was a 3D program that is equipped with a Graphical User Interface (GUI), allowing the user to input various tire parameters and then receive feedback of the resulting wrapping.

# Applied Mathematics & Research Experience (AMRE)

## OARDC

### Tomato Analyzer 4.0

Team members: Nanako Ito, Nan Jiang, Max Taylor, Lydia Kemuma Kinyari
Faculty advisor: Simon Gray (Computer Science)

The OARDC AMRE team was asked to provide ongoing maintenance for Tomato Analyzer, a Windows-based C++ software application originally designed to perform analysis of images of sliced tomato fruits. Over the years Tomato Analyzer's use has expanded to include seeds, leaves, and, most recently, peppers. The algorithms in the initial implementation of Tomato Analyzer assumed that the images to be analyzed were symmetric. However, this assumption has not extended to the kinds of images now being supplied to Tomato Analyzer. The majority of the changes requested dealt with improving the software's ability to deal with asymmetric images. Additional functionality was implemented to deal specifically with peppers. The other task the AMRE team was given was to implement a version of Tomato Analyzer accessible through a web interface.

## Knot Theory

### The Unknotting Problem

Team members: Brian Foley and Michael Bush

Faculty advisors: John Ramsay and Jen Bowen (Math)

The 2015 AMRE Knot Theory Research team worked on 'the unknotting problem.' The unknotting problem is an unsolved problem in knot theory that can be roughly described as follows: Given any mathematical knot - that is, a closed loop of string projected onto the plane with some number of crossings - the unknotting problem is to create an algorithm that determines whether or not that knot can be untangled to a circular loop with no crossings (the unknot), without cutting the string. Our approach to this problem was to design a set of generalized knot moves based on the Reidemeister moves, flypes, and contour moves. We designed an algorithm that intelligently applies these moves in an attempt to reduce the given knot diagram to the unknot. In addition to working on the algorithm, we also worked towards writing a computer program that implements our algorithm to untangle knots using numerical methods as opposed to pencil-and-paper application of our algorithm. Another key component of our work this summer was the development of a mathematical proof that our algorithm is a necessary and sufficient solution to the unknotting problem.

## Brant & Cochran

### Market Research and Business Plan Development for a New Business

Team members: Robert Beall, Jack Marousek, Liam Fukushima
Faculty advisors: Brian Karazsia (Psychology) and Vikki Briggs

Traditionally, AMRE projects have been based in math and computer science; however, opportunities focused on entrepreneurship have been explored recently. Such is the case with Brant and Cochran, an early-stage premium axe manufacturing firm. Most research was delivered in a modified business plan. This consisted of conventional sections such as "Products and Services" and "Marketing Research and Plan," as well as a plan for the coming twelve months. In addition, our team created financial documents and projections for Brant and Cochran, including a four-year pro forma income statement outlining multiple forms the company might take, along with several scenarios within those forms. Competitor's sales and income were also analyzed to estimate the total market for premium axes (as measured by sales revenue and number of axes sold). We also laid the groundwork for the company to begin holding focus groups to finalize core user profiles, a defined product, and brand identity. Our team had a great experience being so exposed to the inner workings of an early-stage startup, local businesses, and the successful individuals who own and operate them.