Time Complexity and Searching

Searching Algorithms

- Can be done one of two ways:
  1. Checking each item to see whether that specific item contained within a data structure (like an array)
  2. Using a key (unique value) to look up an item (like an ID number)

- Very common problem in CS

- For large data structures we need efficient ways to find things

Time Complexity Analysis

- We don't consider the "speed" of an algorithm, but instead the number of steps to takes to accomplish the task.

- We need to know how many steps an algorithm will take based on the size of its input
  - In our cases, we are considering arrays

Searching for an element in an array

- Worst Case
  - The element is NOT in the array
  - Every element must be checked
  - Take n steps (where n is the size of the array)

Array

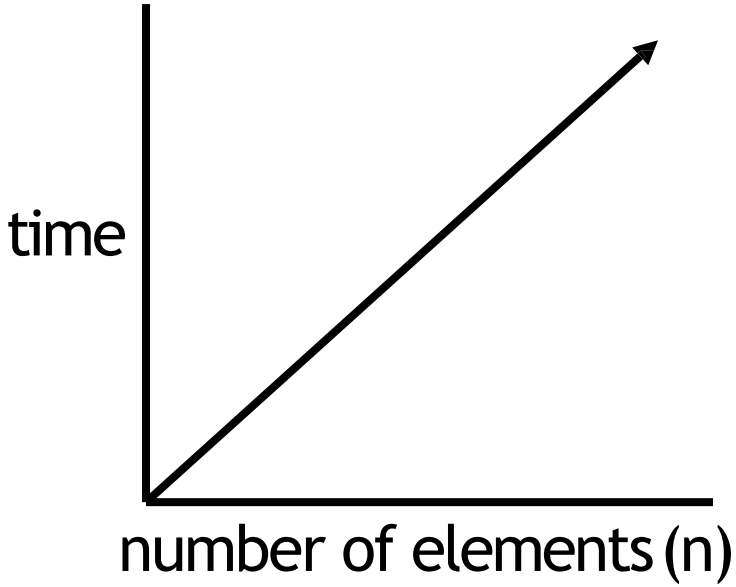
{ 1, 2, -10, 40 ,18}

Searching for an element in an array

- Worst Case
  - The element is NOT in the array
  - Every element must be checked
  - Take n steps (where n is the size of the array)
- The amount of time required to perform the search grows linearly with the size of the input array.

Array

{ 1, 2, -10, 40 ,18}



Searching for an element in an array

- Best Case
  - The element is ALWAYS the first element in the array
  - No elements beyond the first need to be checked

Array

{ 1, 2, -10, 40 ,18}

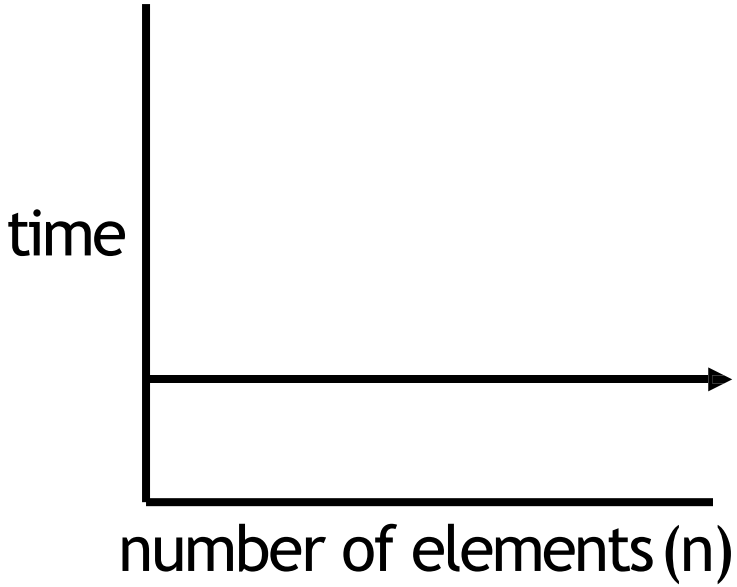
Searching for an element in an array

- Best Case
  - The element is ALWAYS the first element in the array
  - No elements beyond the first need to be checked

- The best-case time is constant no matter the size of the array

Array

{ 1, 2, -10, 40 ,18}



Binary Search

- Only works on a sorted array
- Algorithm:
  - If the array has 0 elements return false
  - Compare the middle element in the array to the target
    - If they are equal, return true
    - If the target is less than the middle element, repeat the search on the elements to the left of the middle
    - If the target is greater than the middle element, repeat the search on the element to the right of the middle

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42}

Search for 42

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40 ,42}

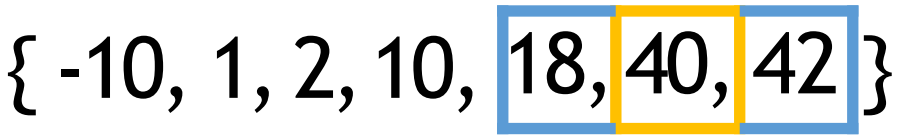
Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40 ,42}

Binary Search Example



Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42}
- Middle is 40, less than 42
- Repeat on { 42 }

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42}
- Middle is 40, less than 42
- Repeat on { 42 }

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40 ,42}
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42 }
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

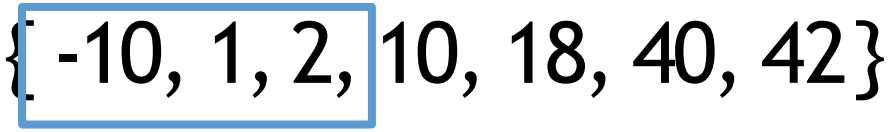
Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42 }
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2 }

Binary Search Example



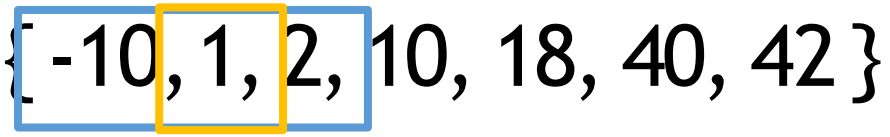
Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42}
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2}

Binary Search Example



Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42}
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2}
- Middle is 1, greater than -30
- Repeat on { -10}

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

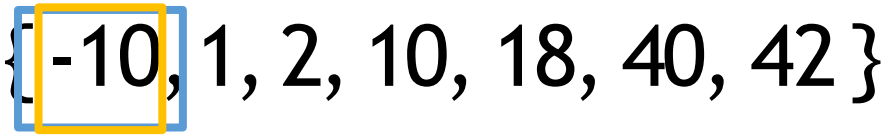
Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42 }
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2 }
- Middle is 1, greater than -30
- Repeat on { -10 }

Binary Search Example



Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42}
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2}
- Middle is 1, greater than -30
- Repeat on { -10}
- Middle is -10, greater than -30
- Repeat on {}

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42 }
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2 }
- Middle is 1, greater than -30
- Repeat on { -10 }
- Middle is -10, greater than -30
- Repeat on {}

Binary Search Example

{ -10, 1, 2, 10, 18, 40, 42 }

Search for 42

- Middle is 10, less than 42
- Repeat on { 18, 40, 42 }
- Middle is 40, less than 42
- Repeat on { 42 }
- Middle is 42, return true

Search for -30

- Middle is 10, greater than -30
- Repeat on { -10, 1, 2 }
- Middle is 1, greater than -30
- Repeat on { -10 }
- Middle is -10, greater than -30
- Repeat on {}
- Empty array, return false

Binary Search in Code

0	1	2	3	4	5	6	7	8
-40	-23	5	10	27	33	35	41	42

bool binary_search(int target, int *array, size_t size)

- size = 9

- middle = size / 2 = 4
  - Odd array size give a true middle
  - Even gives a value slightly left of middle

Binary Search in Code

0	1	2	3	4	5	6	7	8
-40	-23	5	10	27	33	35	41	42

bool binary_search(int target, int *array, size_t size)

- Searching for 5
  - Middle (index 4) is 27, so we need to search the left half.
  - return binary_search(target, array, middle)

- Next Search:

0	1	2	3
-40	-23	5	10

Binary Search in Code

0	1	2	3	4	5	6	7	8
-40	-23	5	10	27	33	35	41	42

bool binary_search(int target, int *array, size_tsize)

- Searching for 41
  - Middle (index 4) is 27, so we need to search the right half.
  - return binary_search(target, array + middle + 1, size - middle - 1)

- Next Search:

5	6	7	8
33	35	41	42

Binary Search Worst Case Analysis

	Size	# of Calls to Binary Search (worst case)
2^3	8	5

Binary Search Worst Case Analysis

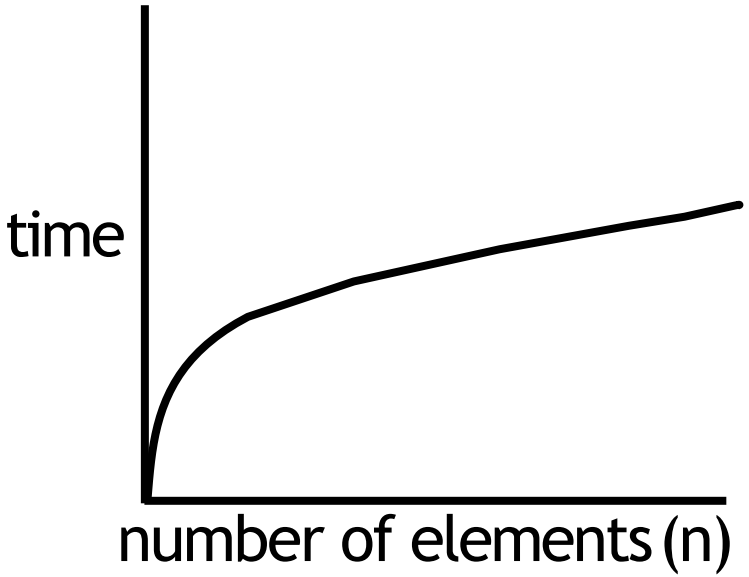
	Size	# of Calls to Binary Search (worst case)
2^3	8	5
2^4	16	6
2^5	32	7

Binary Search Worst Case Analysis

	Size	# of Calls to Binary Search (worst case)
2^3	8	5
2^4	16	6
2^5	32	7
$2^{(\log_2(n))}$	n	$\log_2(n) + 2$

Binary Search Worst Case Analysis

	Size	# of Calls to Binary Search (worst case)
2^3	8	5
2^4	16	6
2^5	32	7
$2^{(\log_2(n))}$	n	$\log_2(n) + 2$



Logarithmic Time Complexity!!!!