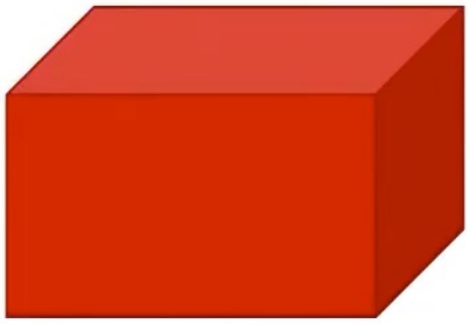
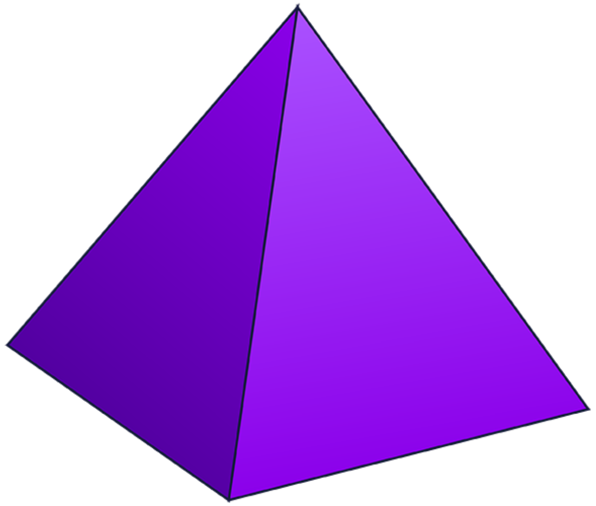
# Viewing

# So far…

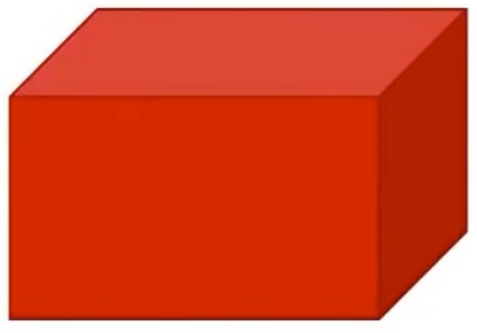
Focused on creating 3D geometric shapes





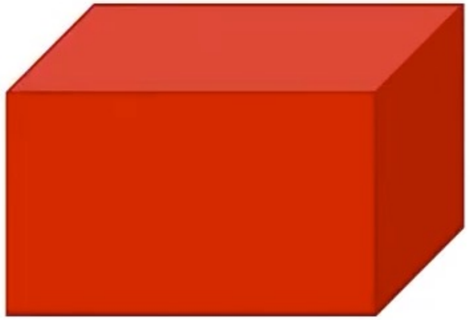
# The Goal

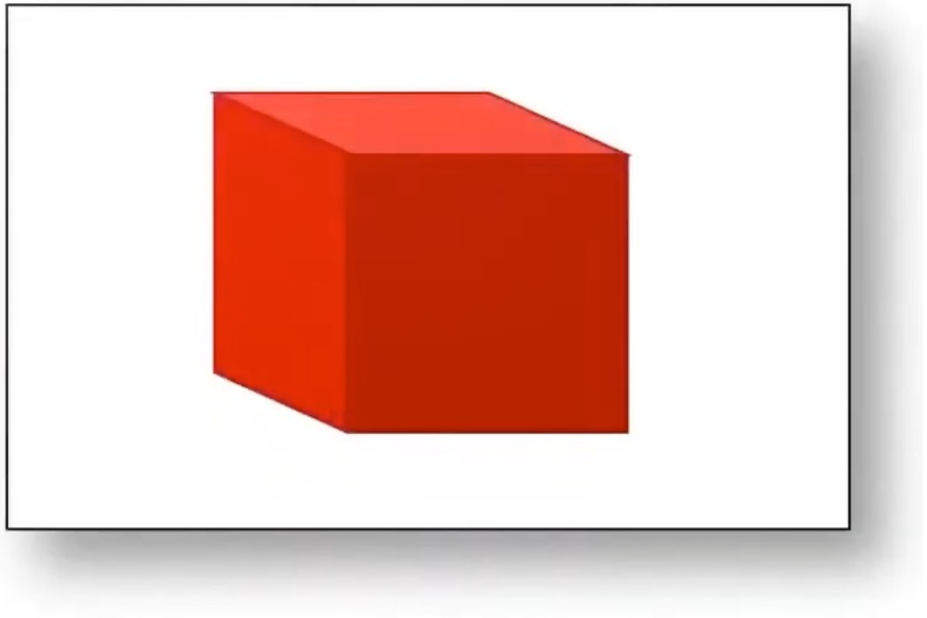
Turn that object into a flat image



# The Goal

Turn that object into a flat image





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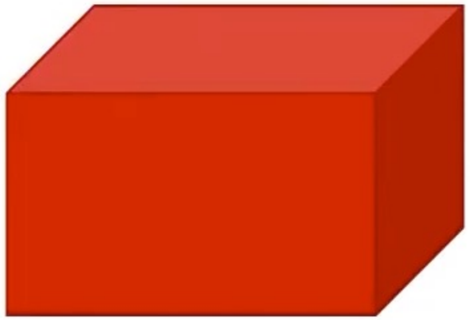
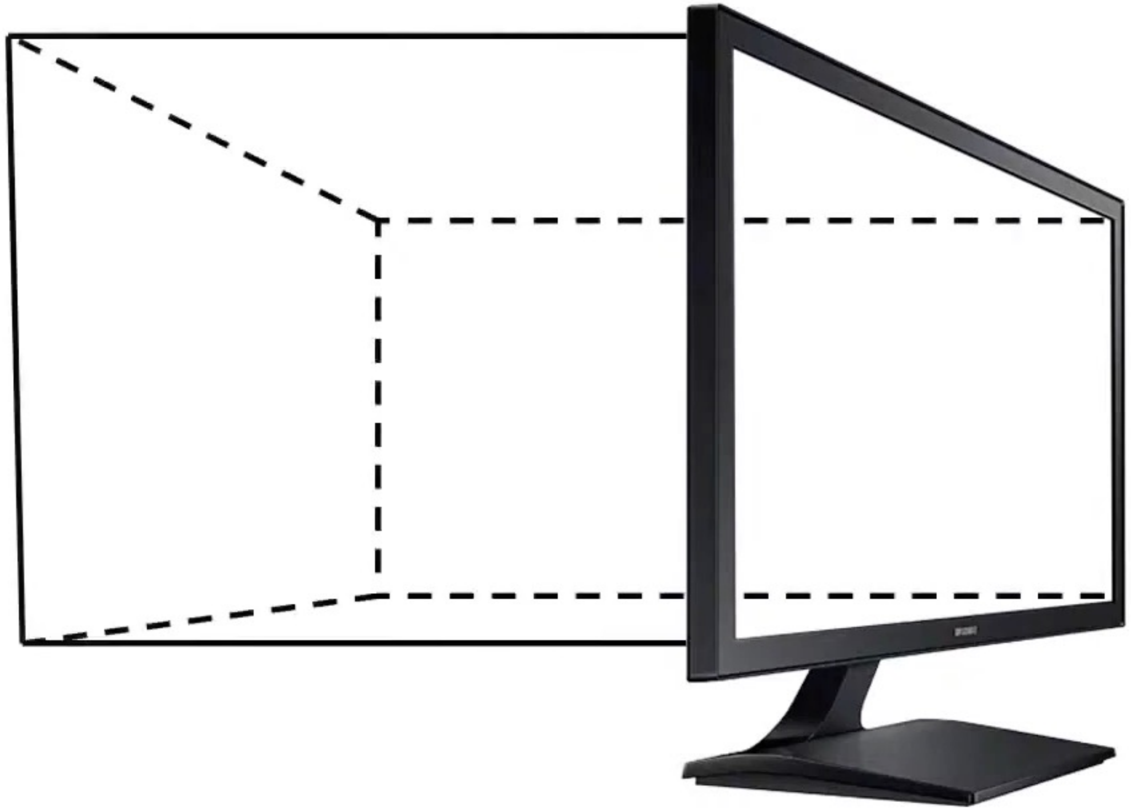
Turn that object into a flat image



# The Concept

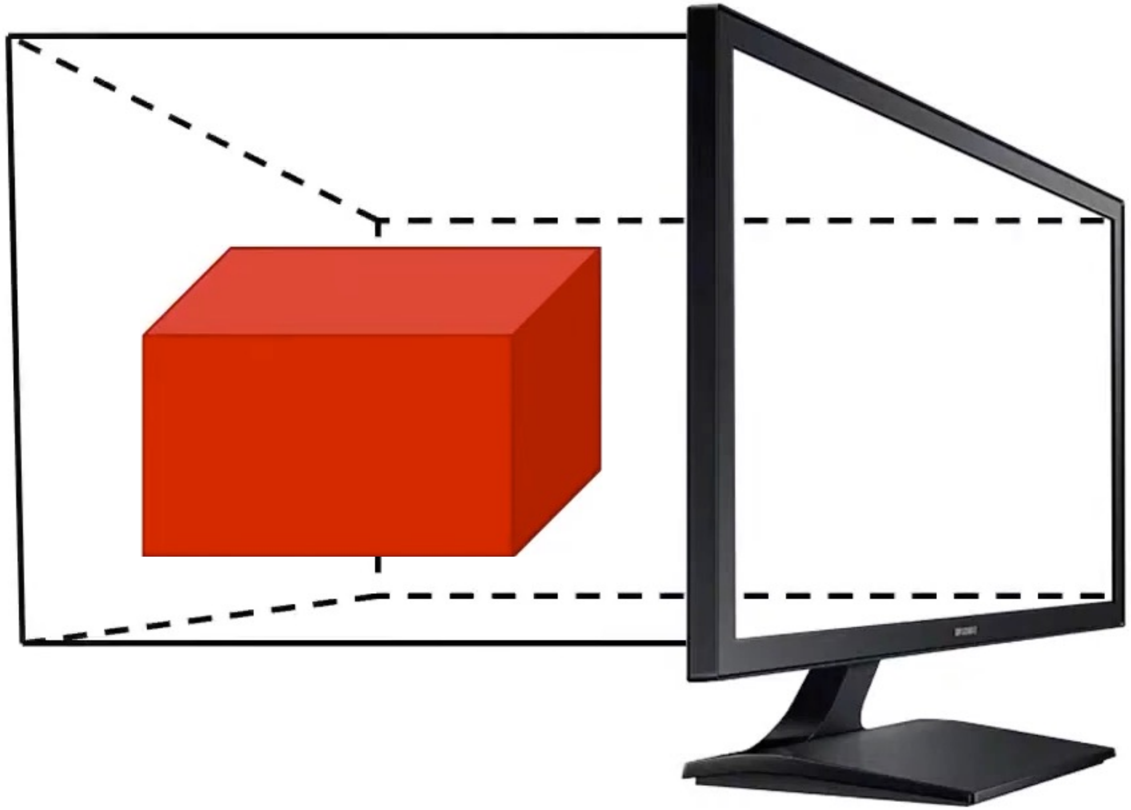
- We have a display

# The Concept



- We have a display
- Acts like a window into our scene
- The scene is behind the display

# The Concept



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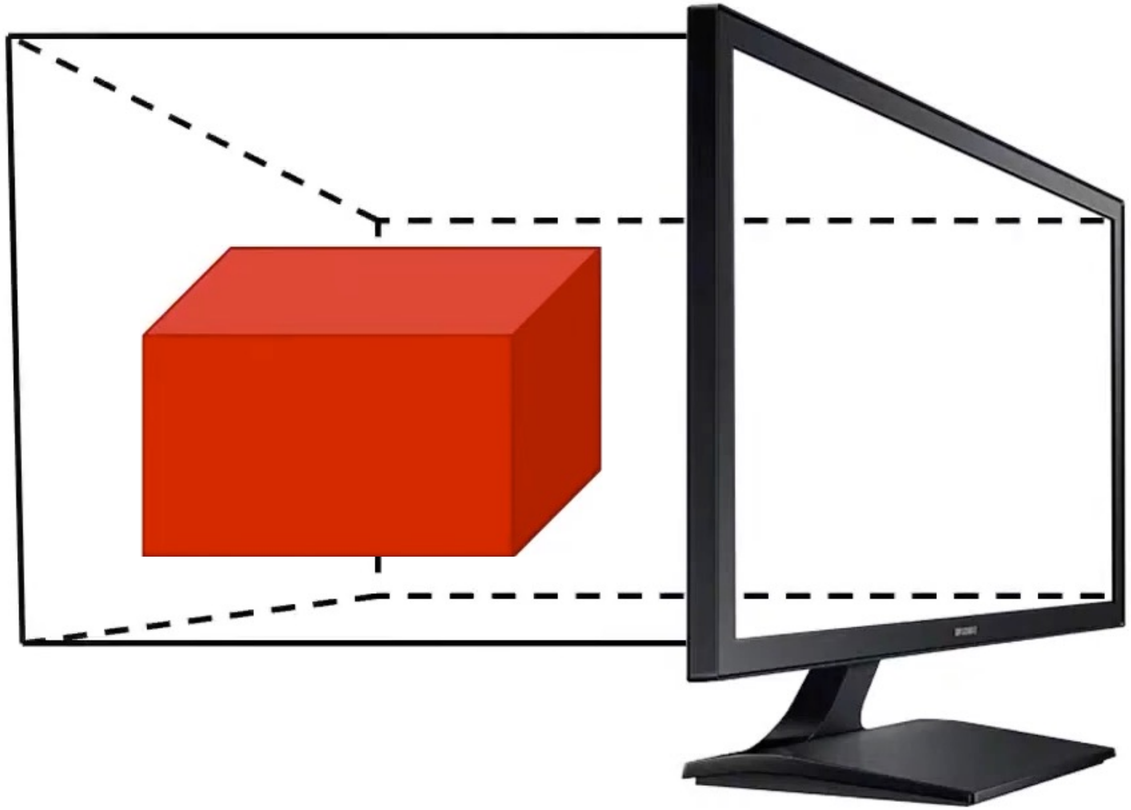
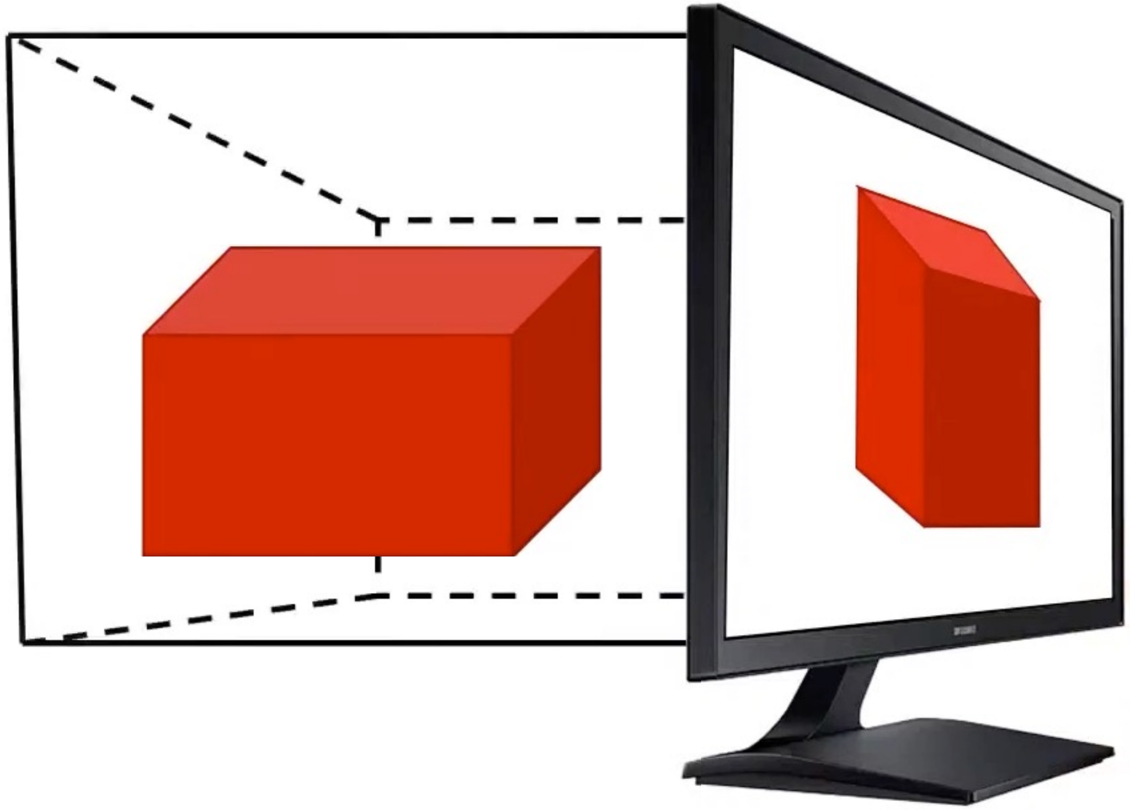


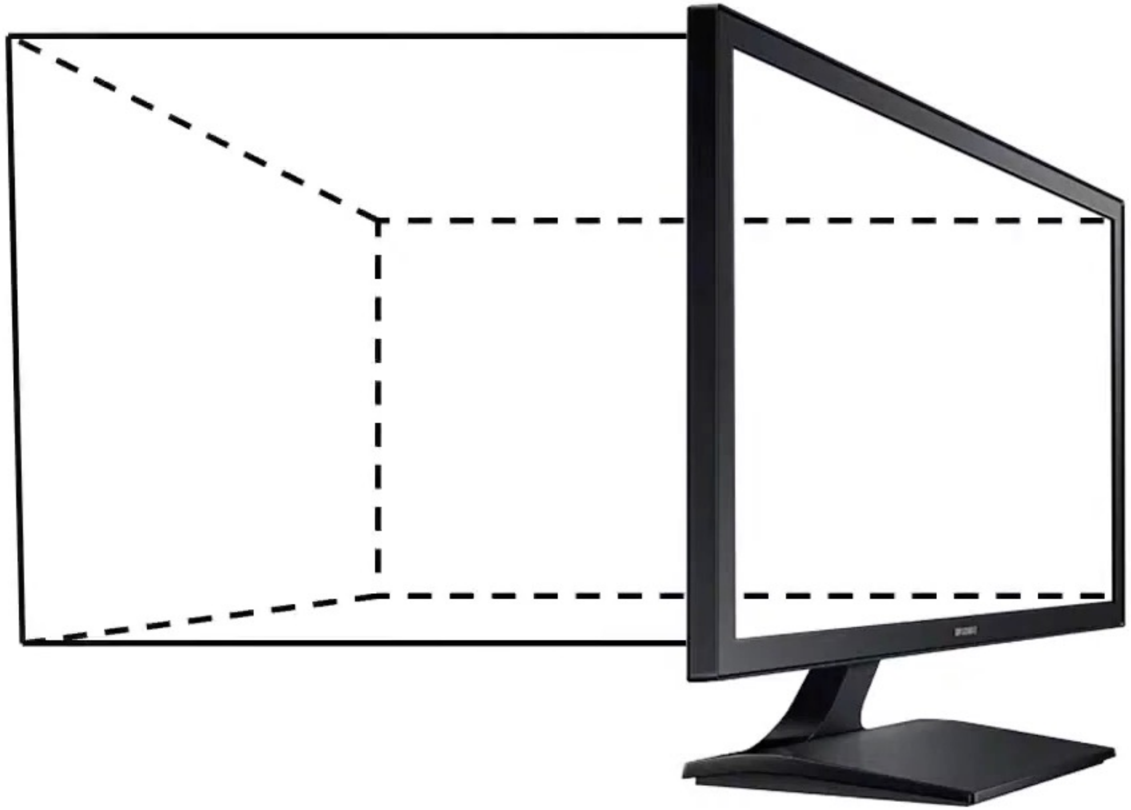
- We have a display
- Acts like a window into our scene
- The scene is behind the display
- Project the object onto the display

# The Concept



- We have a display
- Acts like a window into our scene
- The scene is behind the display
- Project the object onto the display
- How?

# A New Coordinate Frame



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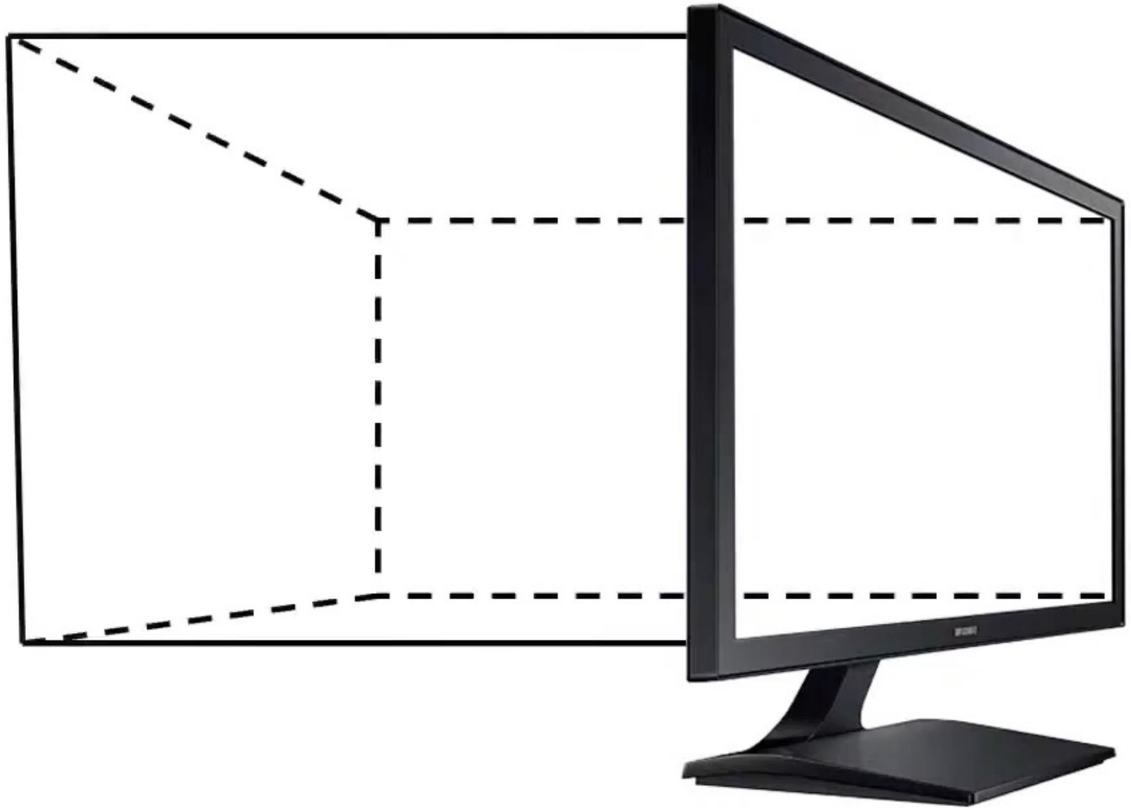

- Need to identify an origin

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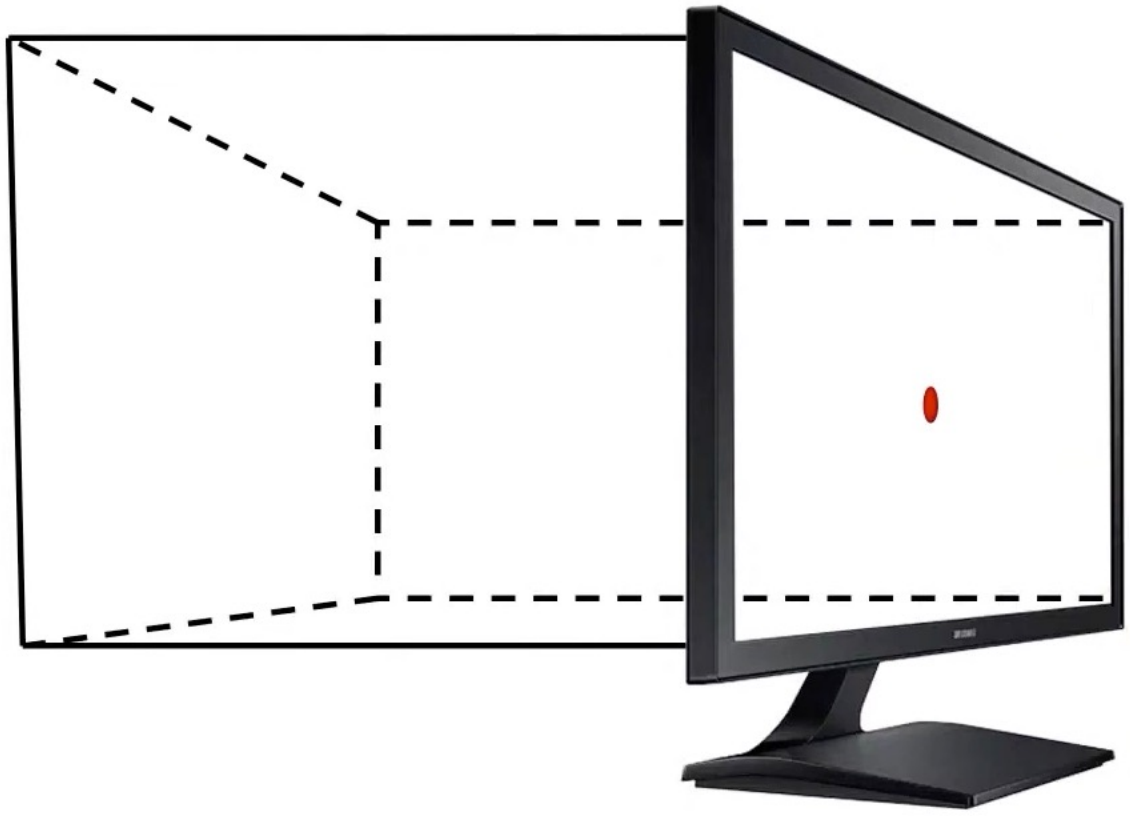
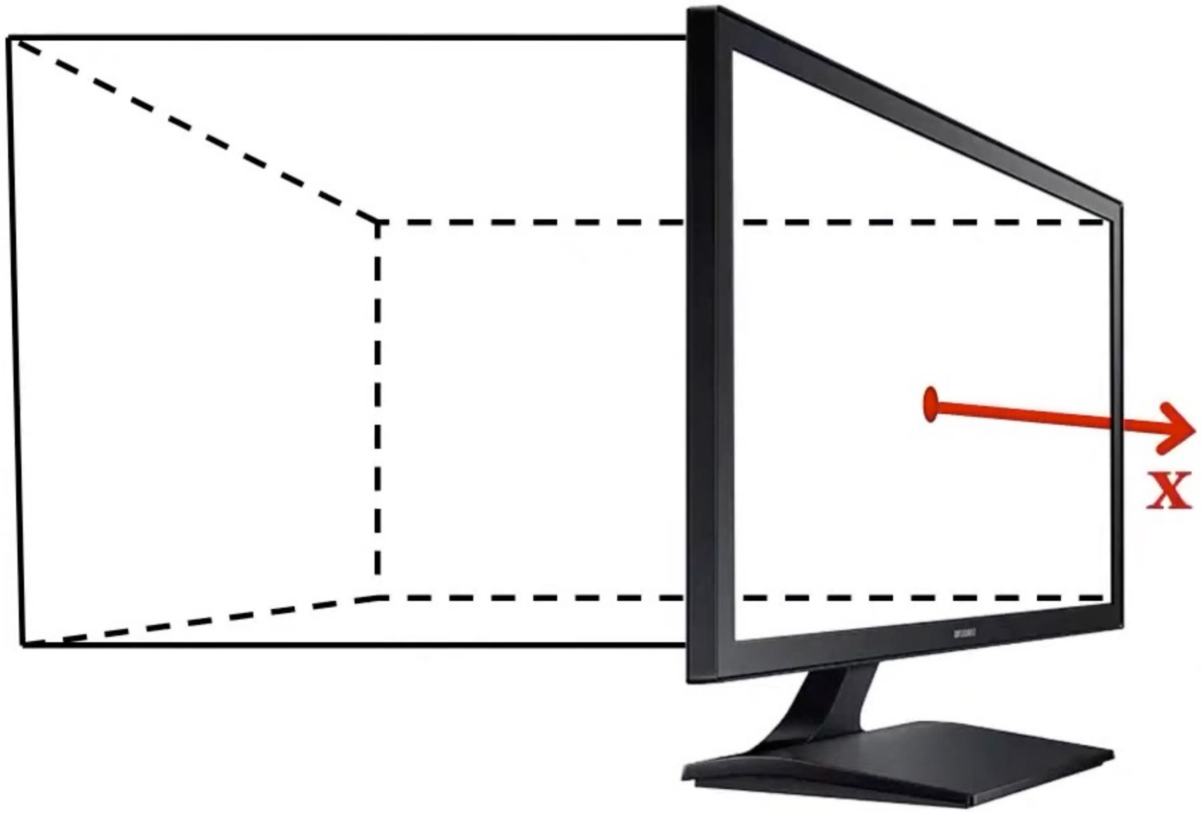


- Need to identify an origin
- Need basis vectors

# A New Coordinate Frame

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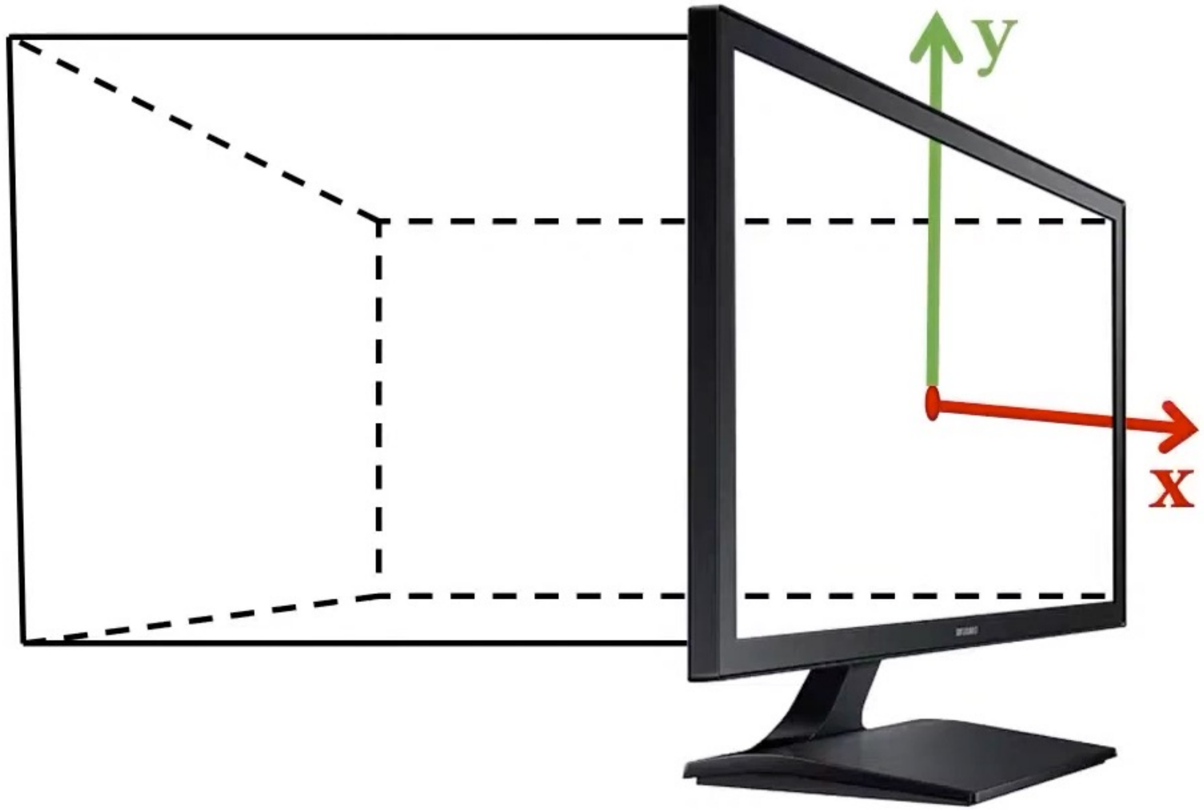


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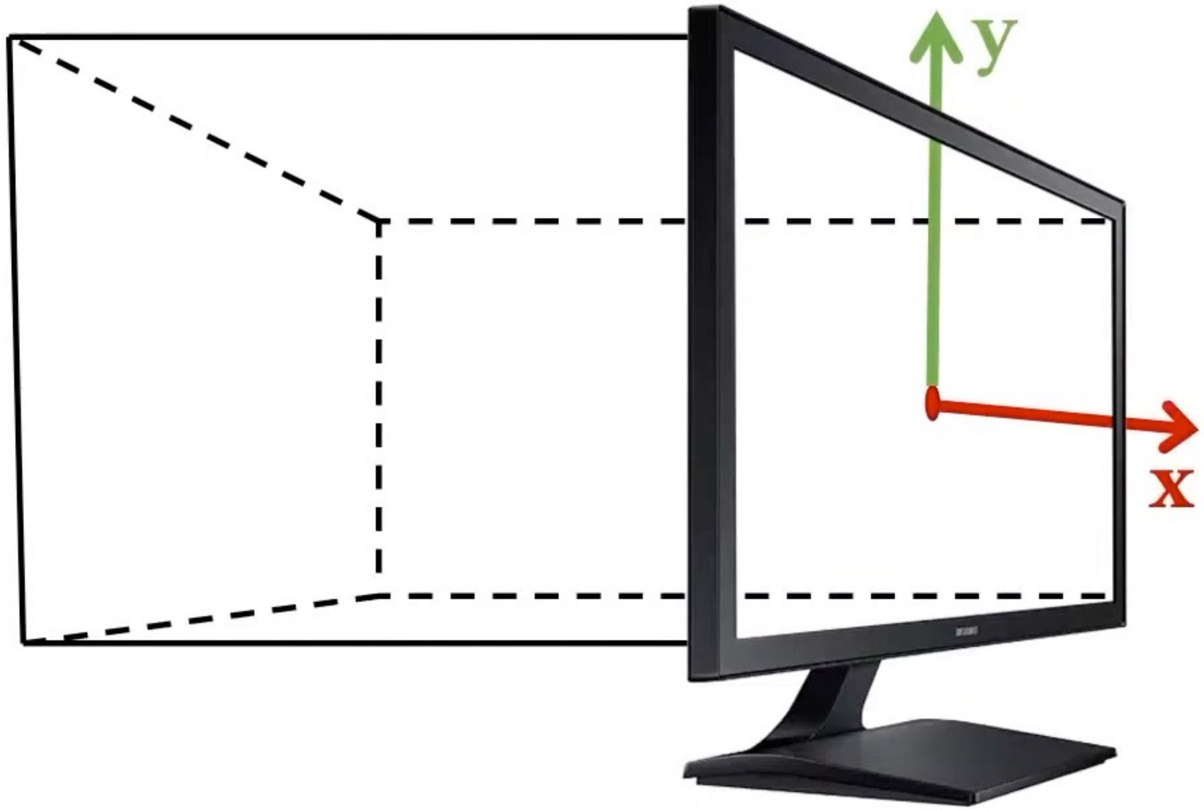


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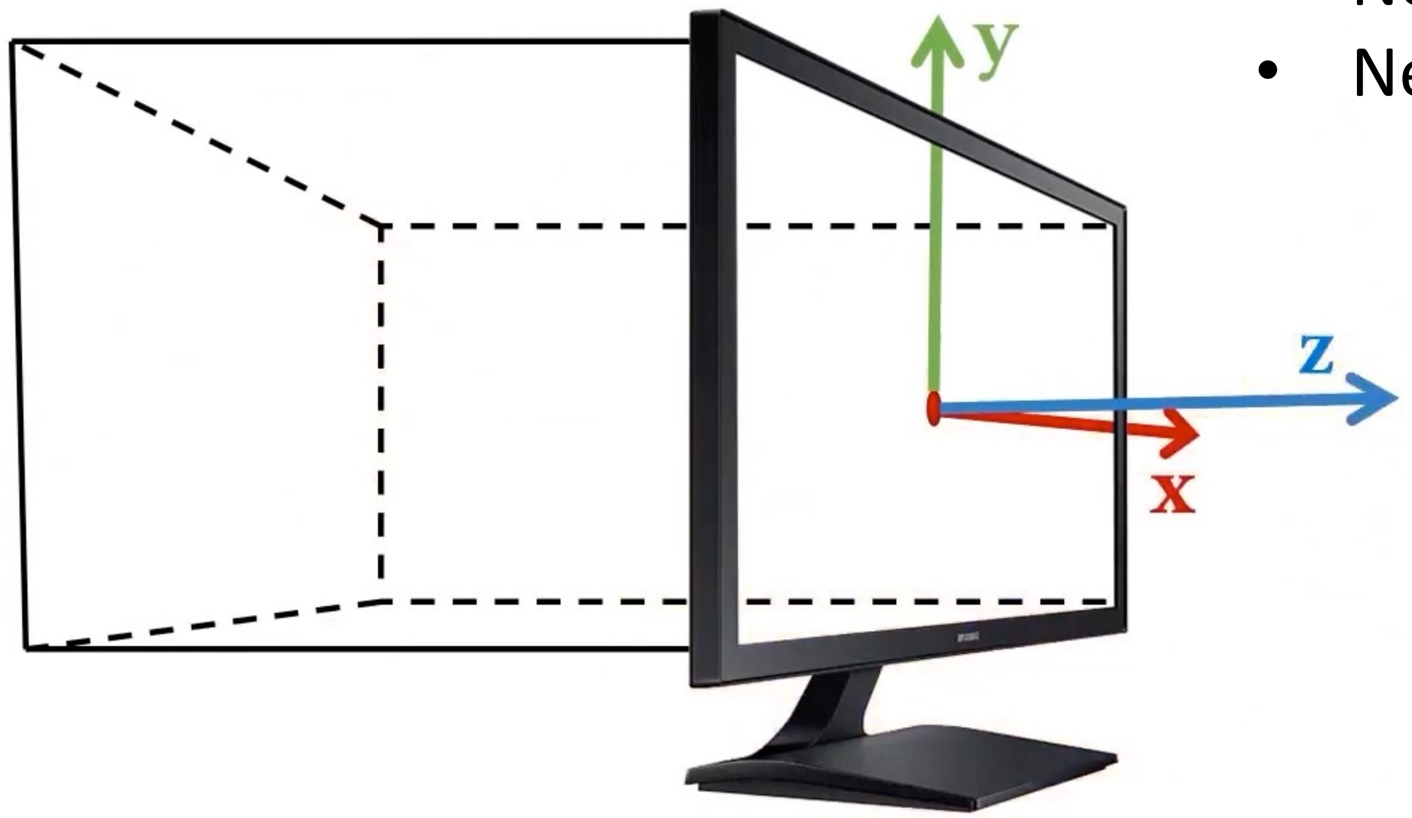
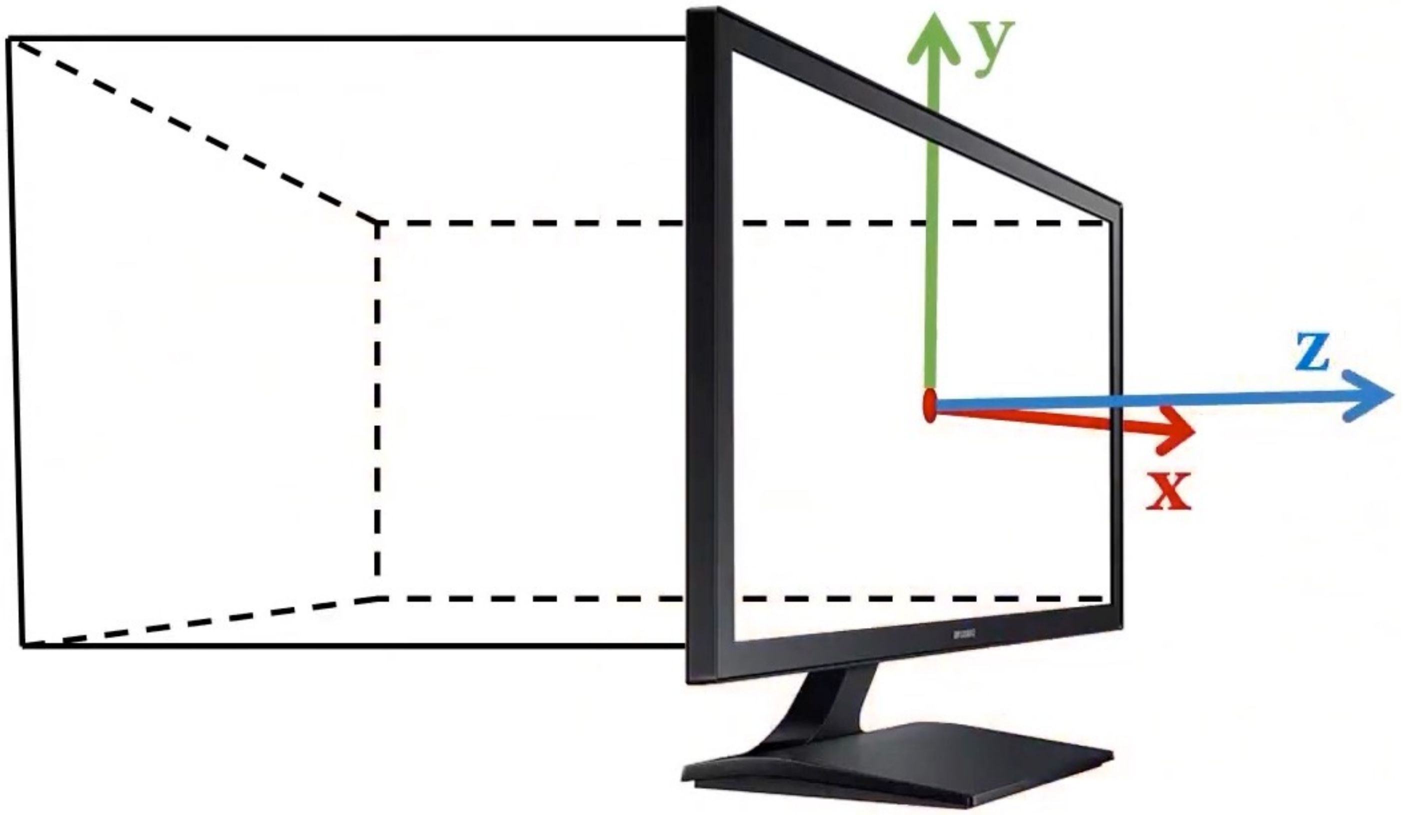
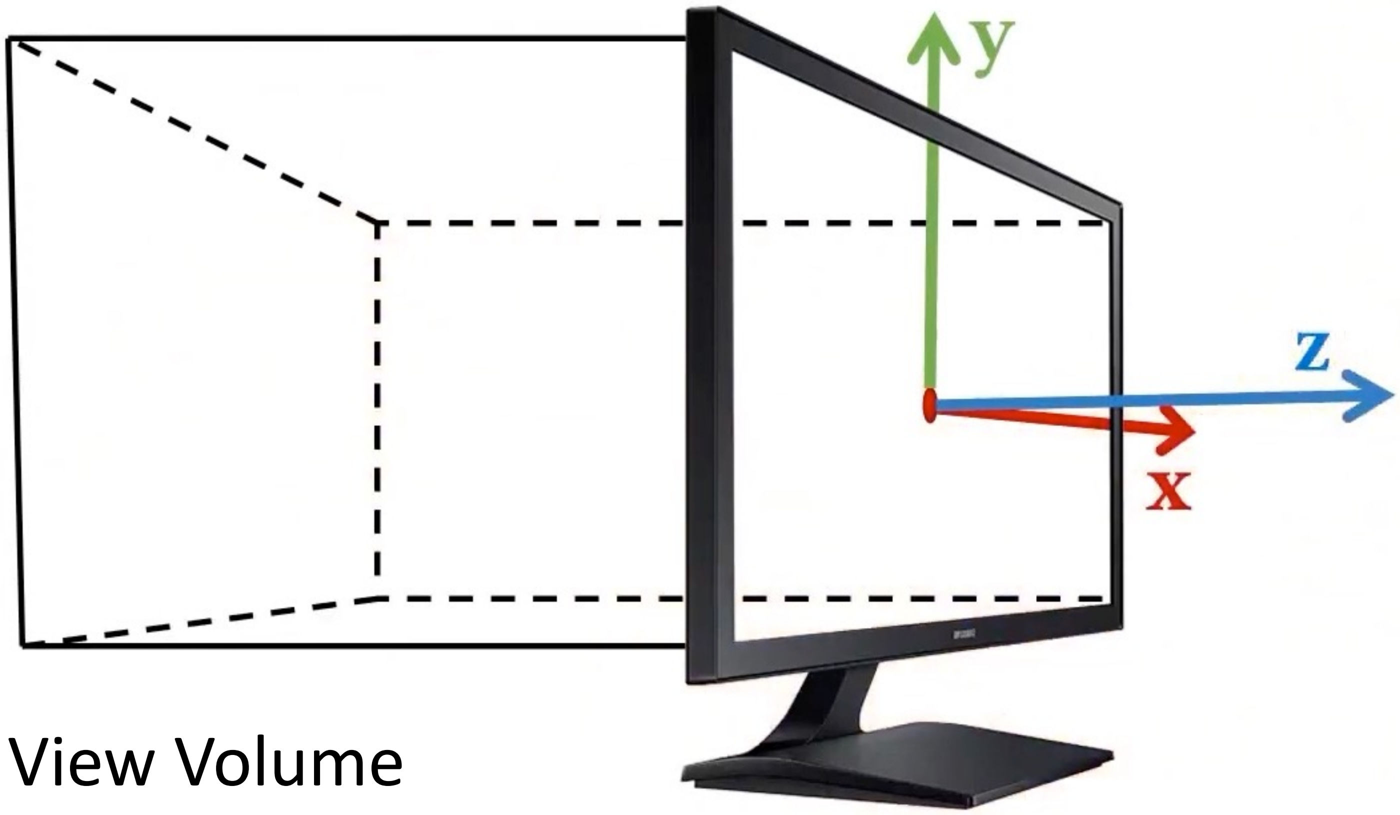



- Need to identify an origin
- Need basis vectors

# The Eye/Camera Coordinate Frame

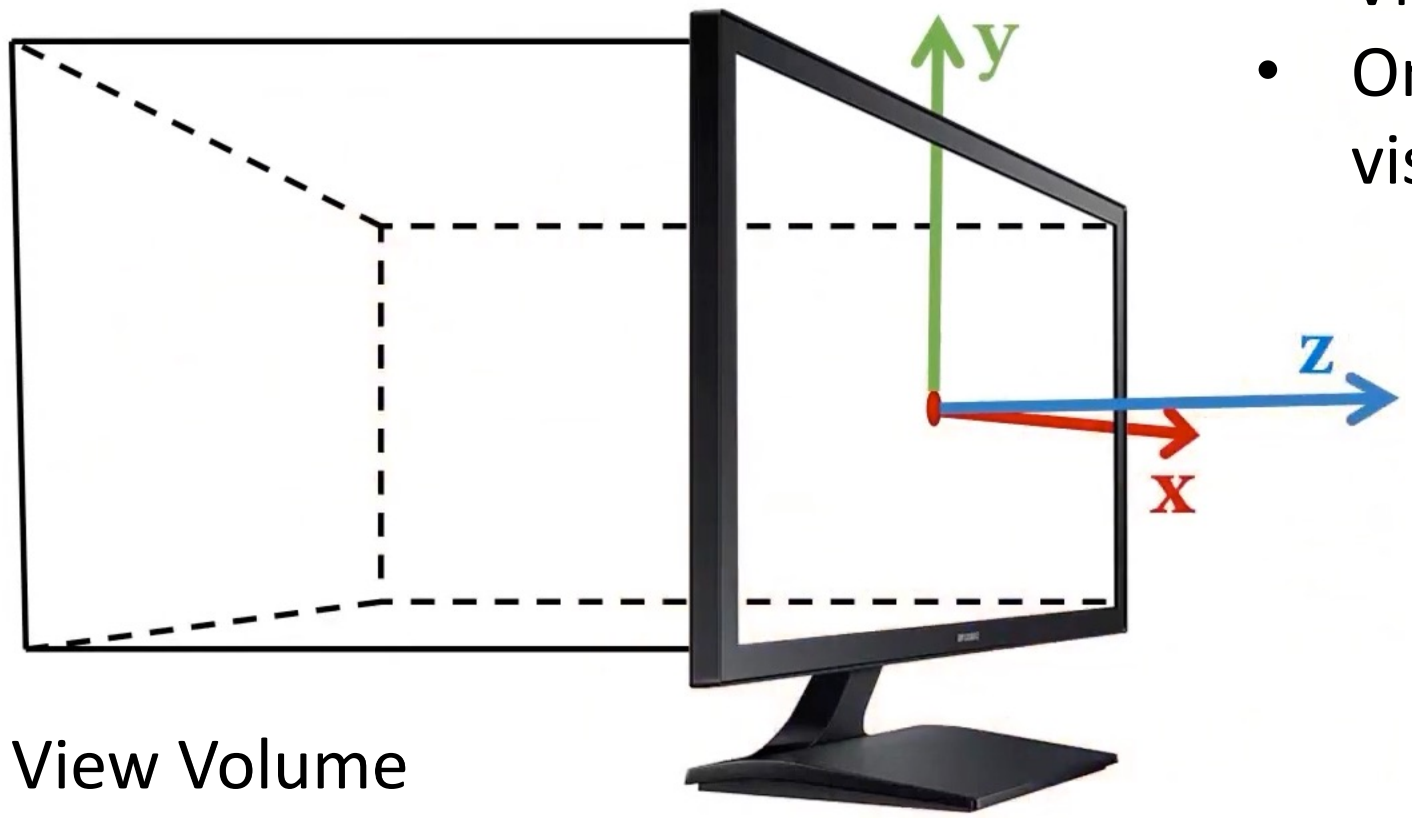


# The Eye/Camera Coordinate Frame




View Volume

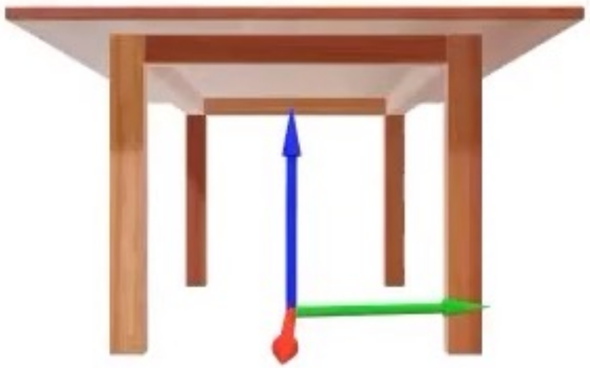
# The Eye/Camera Coordinate Frame




View Volume

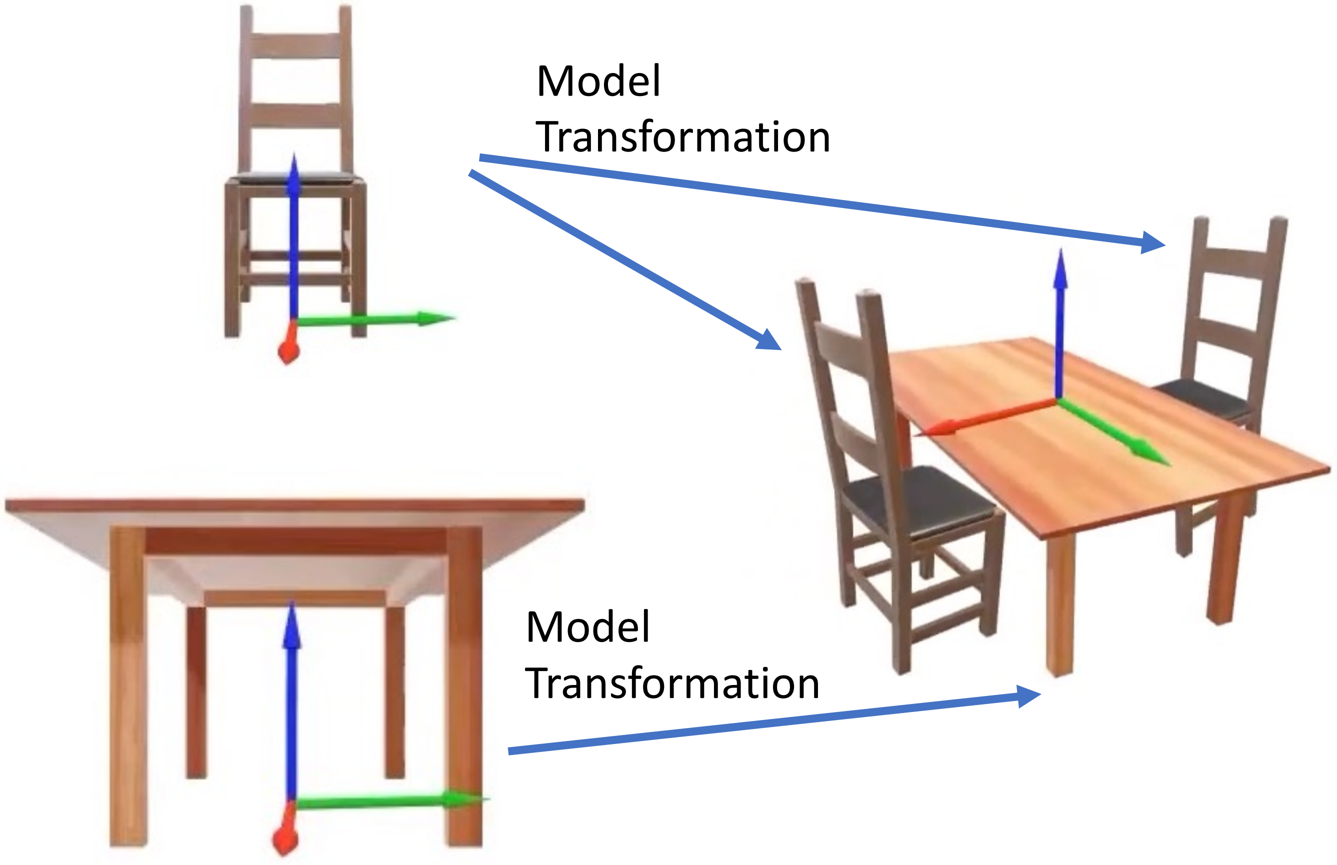
- View volume is what your can see
- Only things in the view volume are visible

# View Transformations



Model Frame

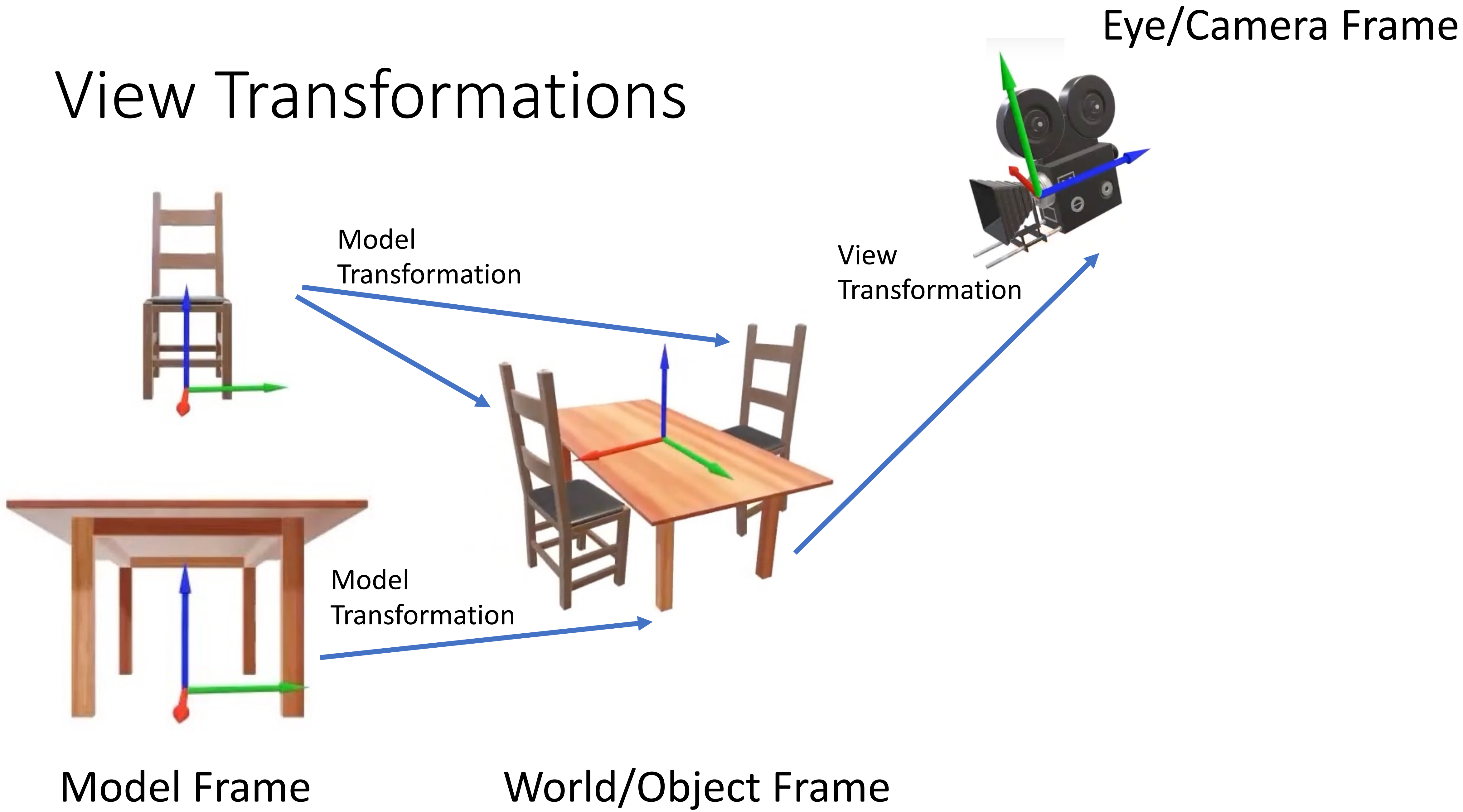
# View Transformations



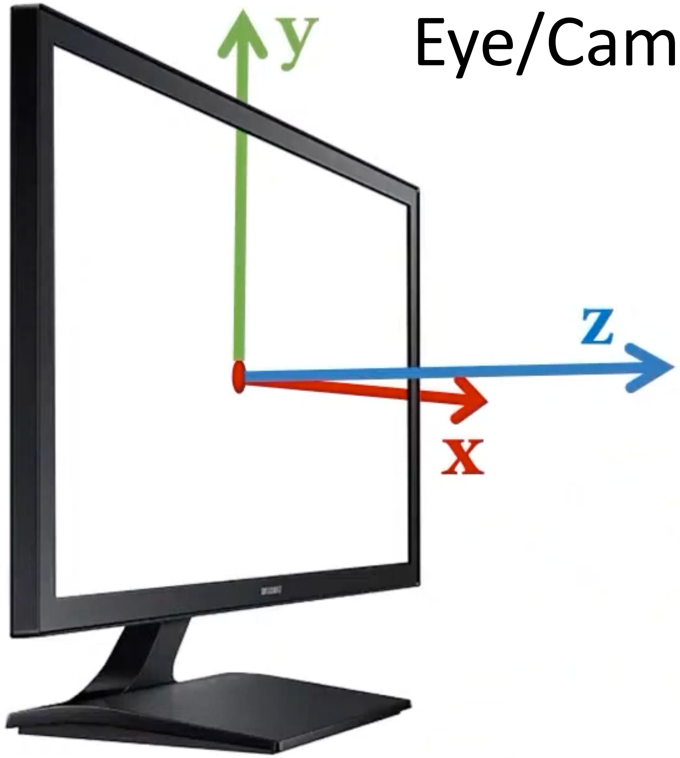

Model Frame

World/Object Frame

# View Transformations

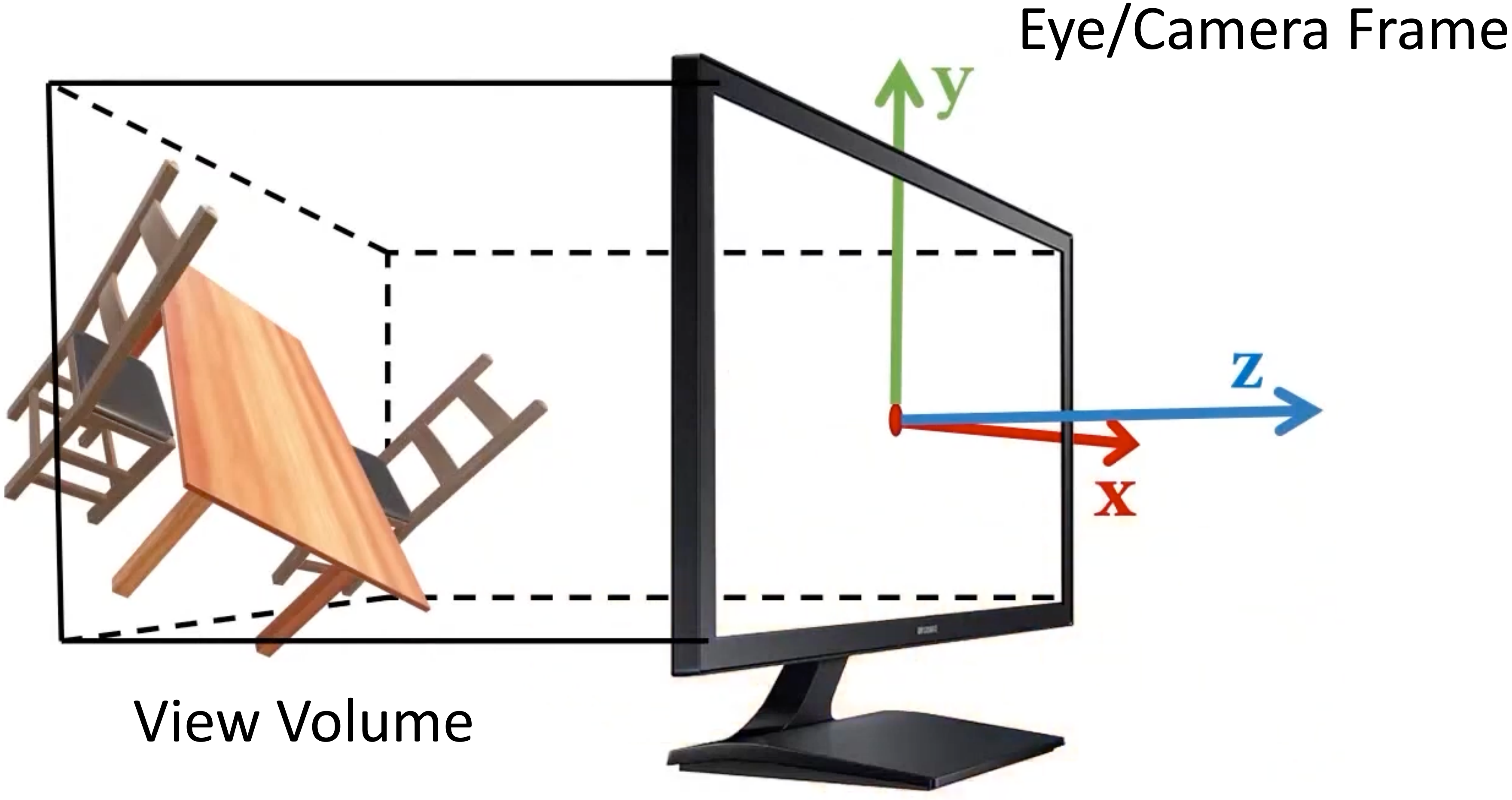


# Projection

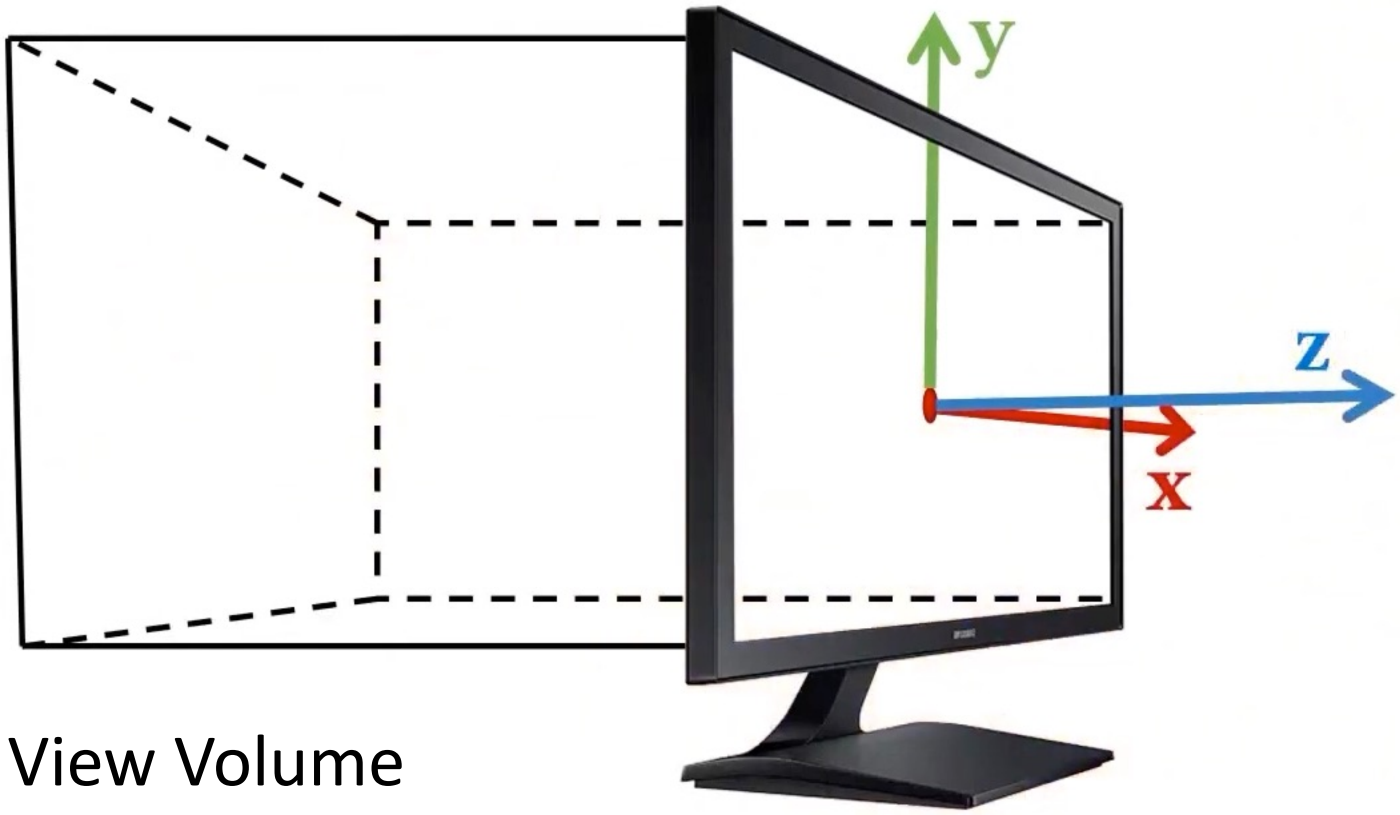



Eye/Camera Frame

# Projection

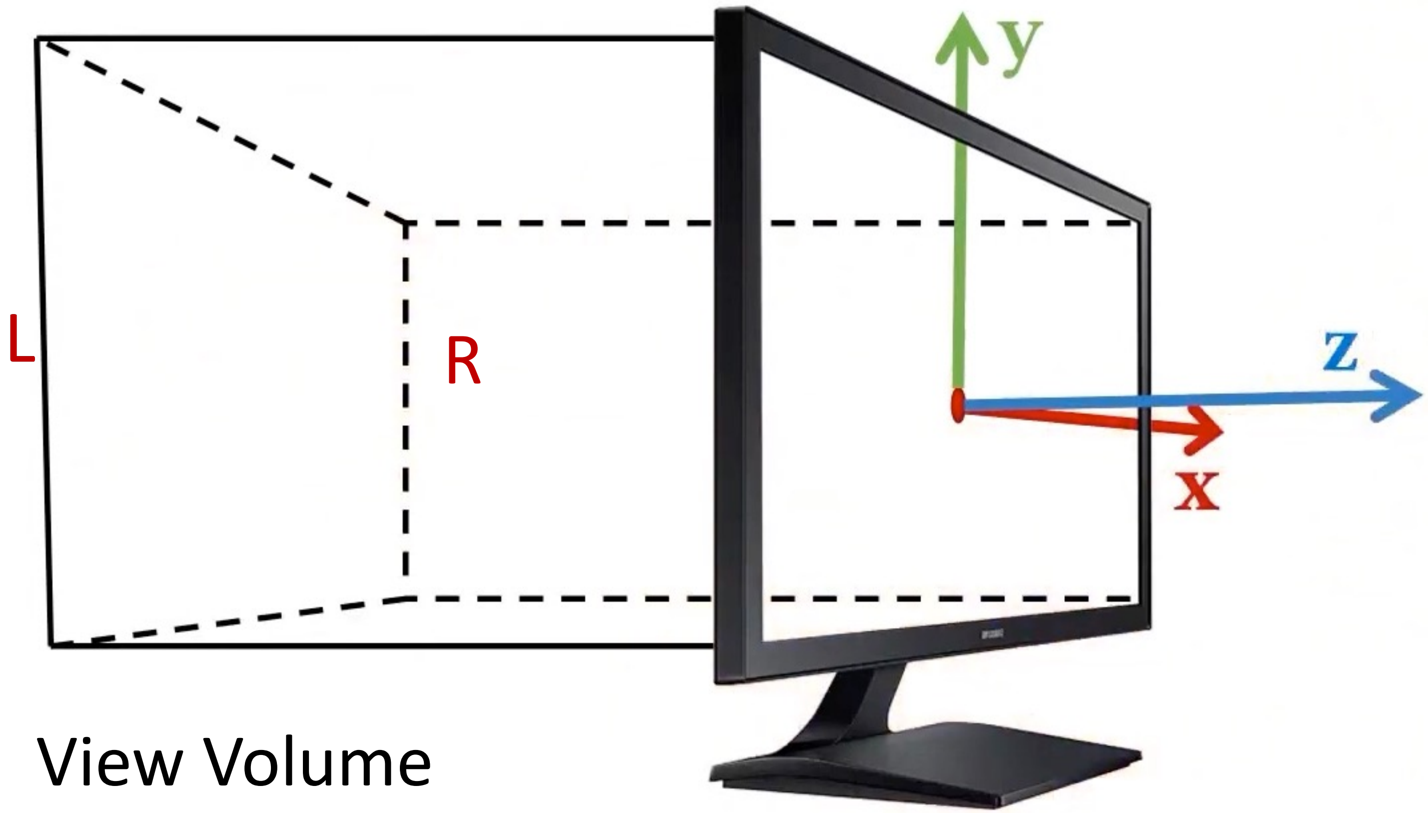


# Defining the View Volume




View Volume

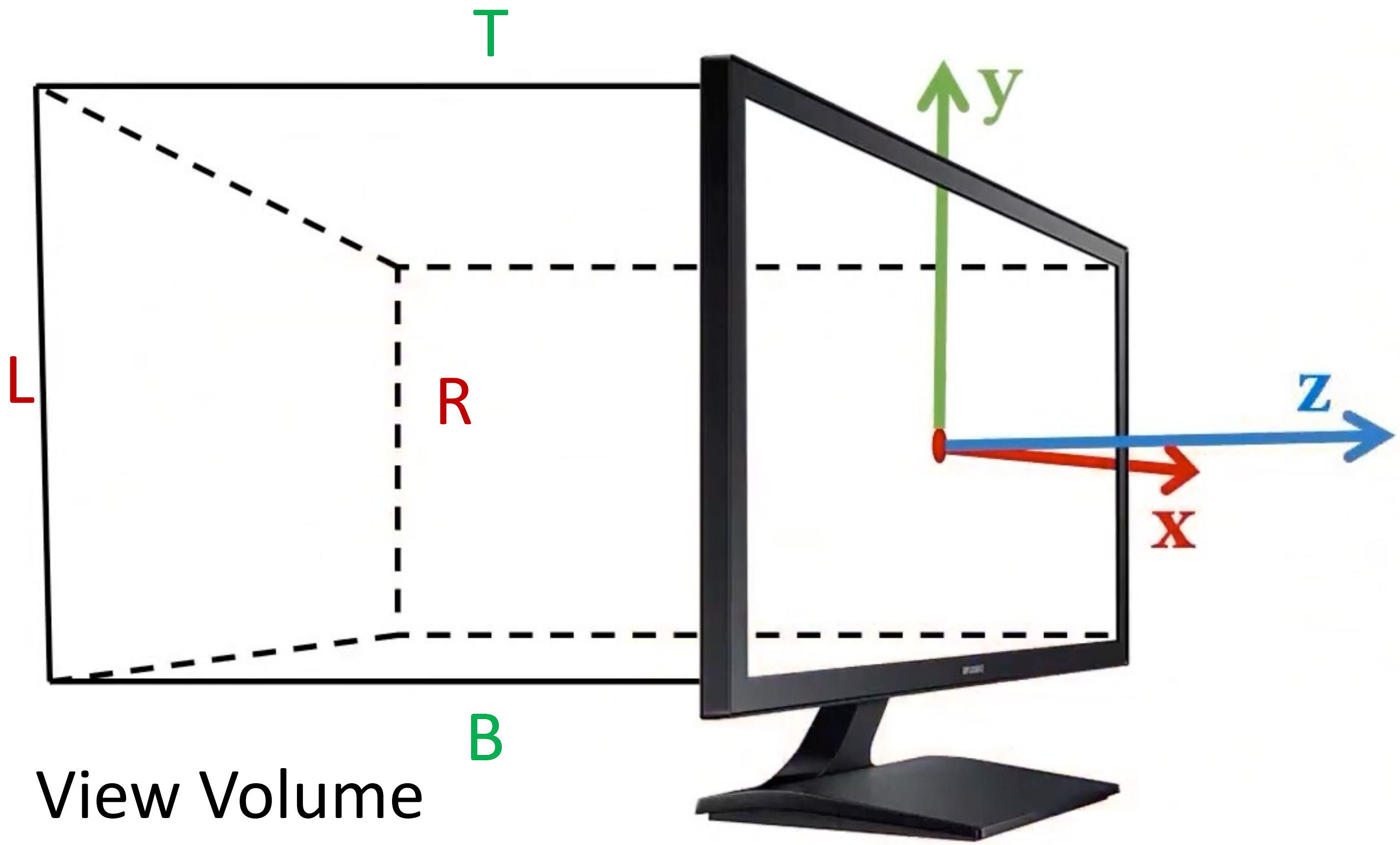
# Defining the View Volume




View Volume

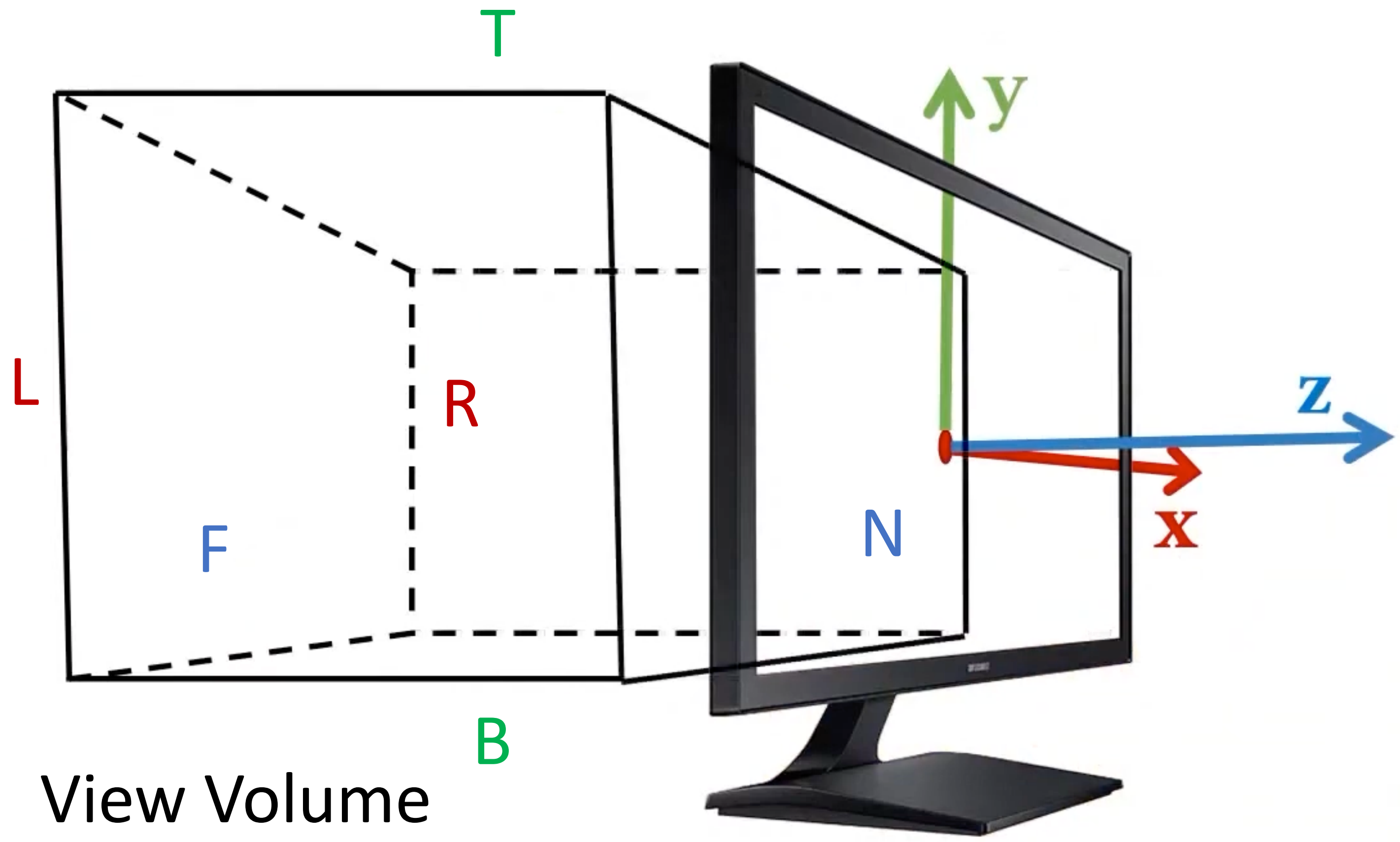
- Left and Right (X values)

# Defining the View Volume



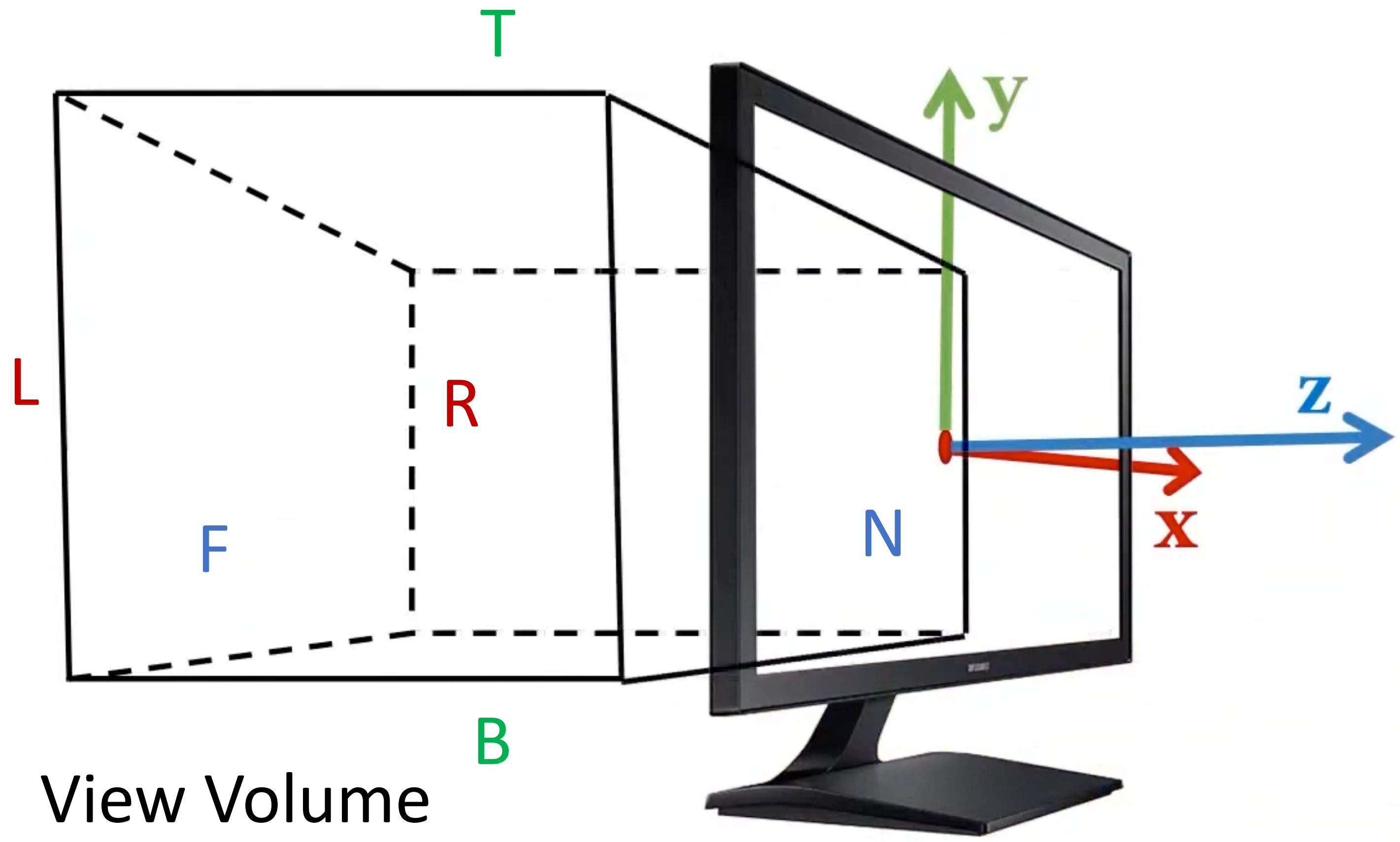

- Left and Right (X values)
- Top and Bottom (Y Values)

# Defining the View Volume




- Left and Right (X values)
- Top and Bottom (Y Values)
- Near and Far (Z Values)
  - In most cases Near is **NOT** Zero

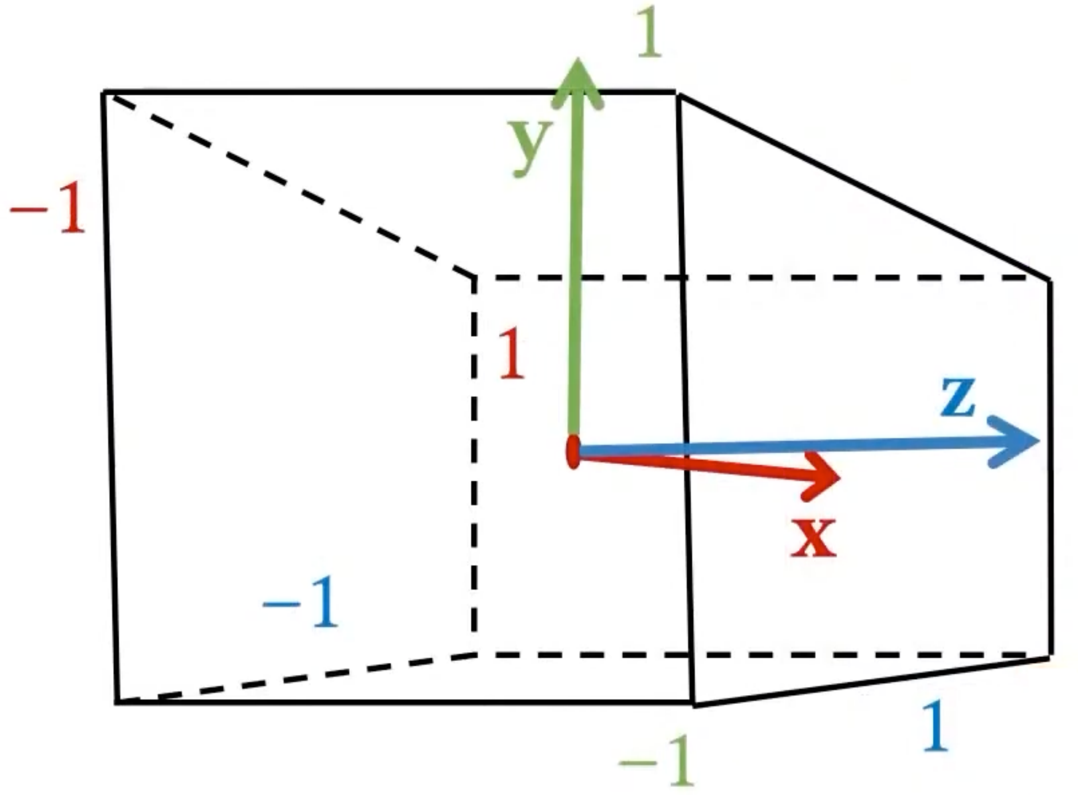
# Defining the View Volume




View Volume

- Left and Right (X values)
- Top and Bottom (Y Values)
- Near and Far (Z Values)
  - In most cases Near is **NOT** Zero
- Want to convert this to a more standard coordinate system

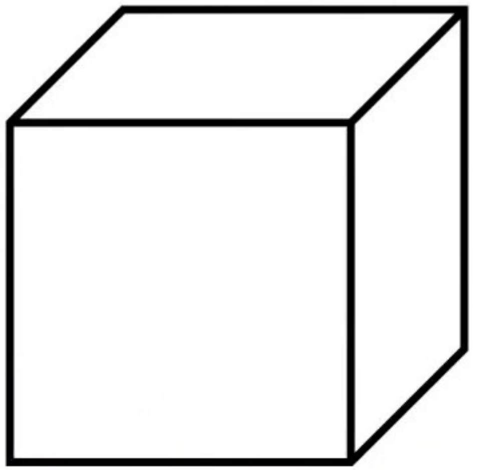
# Canonical View Volume




- All directions go from -1 to 1
- Origin is center of the cube
  - 2x2x2
- Want to convert from Eye/Camera Frame to the Canonical View Volume
- Need a projection transformation

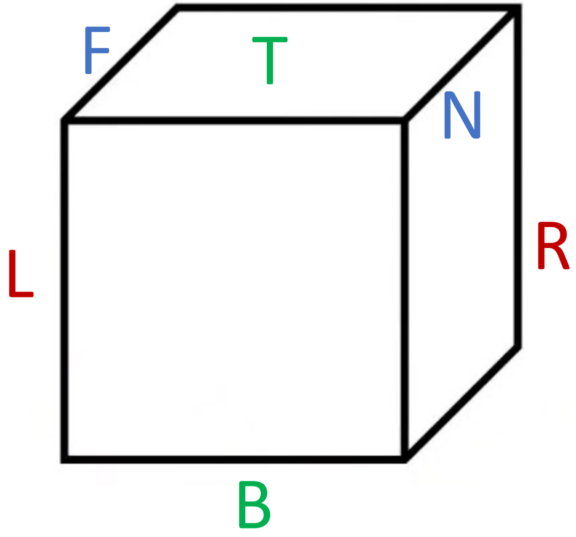
# Orthographic Projection

# Orthographic Projection



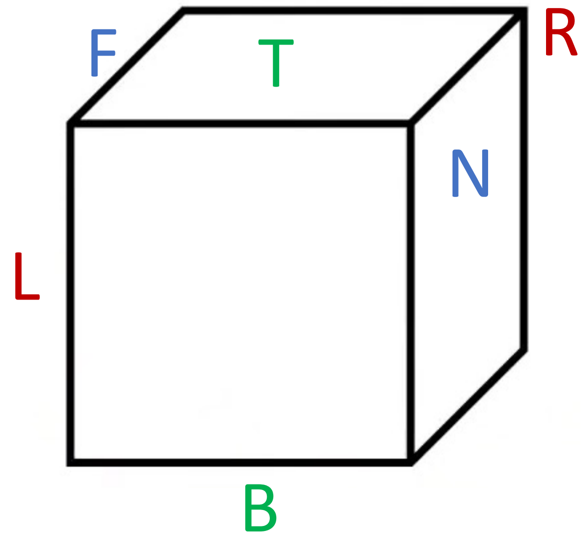
View/Camera

# Orthographic Projection



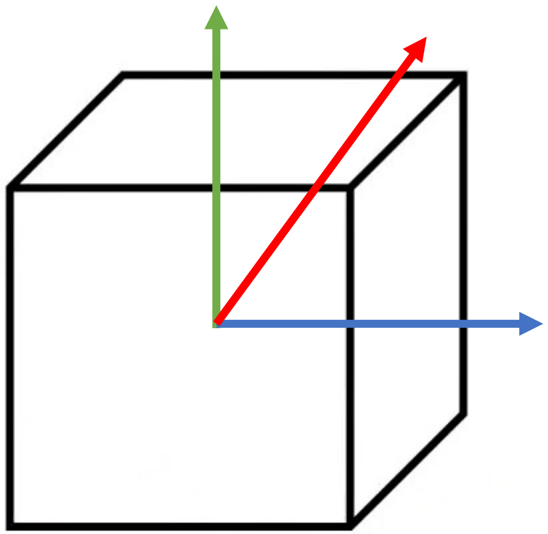

View/Camera

# Orthographic Projection



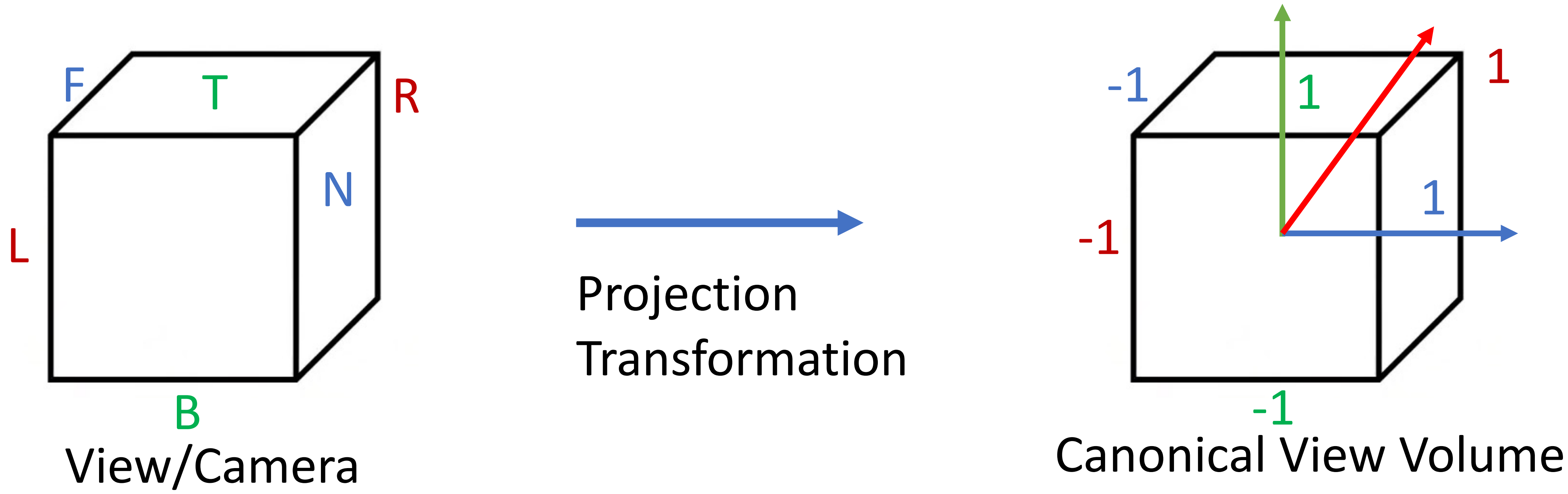
View/Camera

Projection
Transformation



Canonical View Volume

# Orthographic Projection



# Orthographic Projection Matrix

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Orthographic Projection Matrix

Canonical View Volume Frame

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Camera Frame

# Orthographic Projection Matrix

Canonical View Volume Frame

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Camera Frame

# Orthographic Projection Matrix

$$\underset{\text{Canonical View Volume Frame}}{\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix}} = \begin{pmatrix} ? & 0 & 0 & ? \\ 0 & ? & 0 & ? \\ 0 & 0 & ? & ? \\ 0 & 0 & 0 & 1 \end{pmatrix} \underset{\text{Camera Frame}}{\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}}$$

- No rotation
- Need scaling
- Need Transformation

# Orthographic Projection Matrix

Canonical View Volume Frame

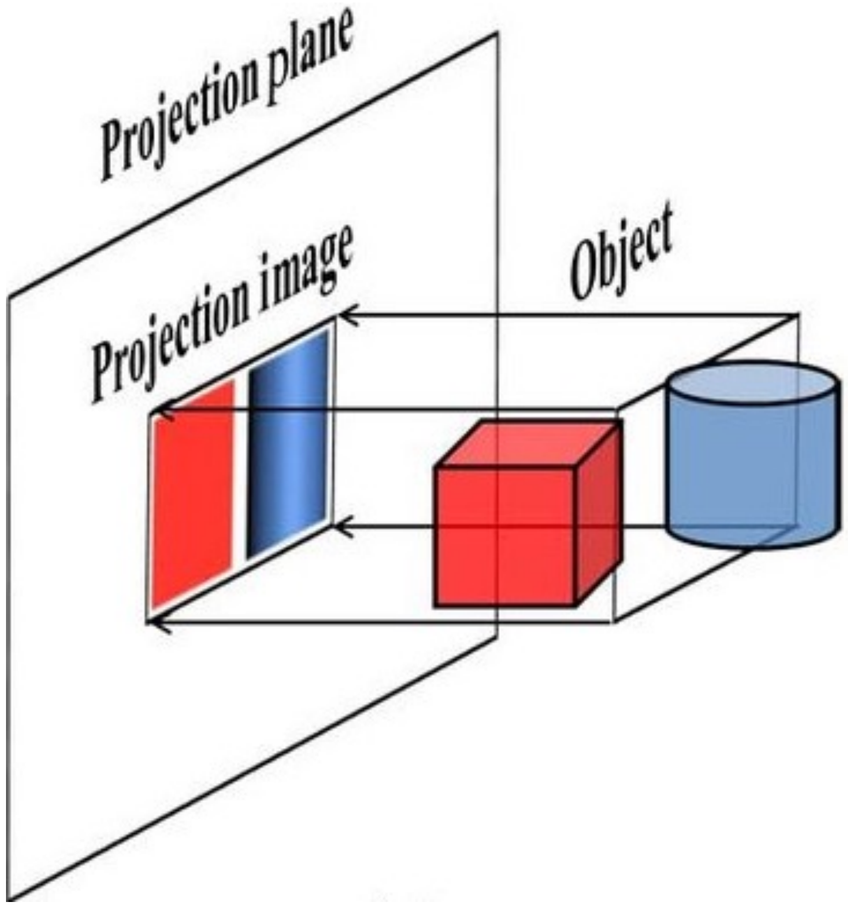
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{l+r}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Camera Frame

- No rotation
- Need scaling
  - Normalize the values to fit into the range (-1 to 1)
- Need transformation
  - Move the camera origin to the center of the view volume

# Properties of Orthographic Projection

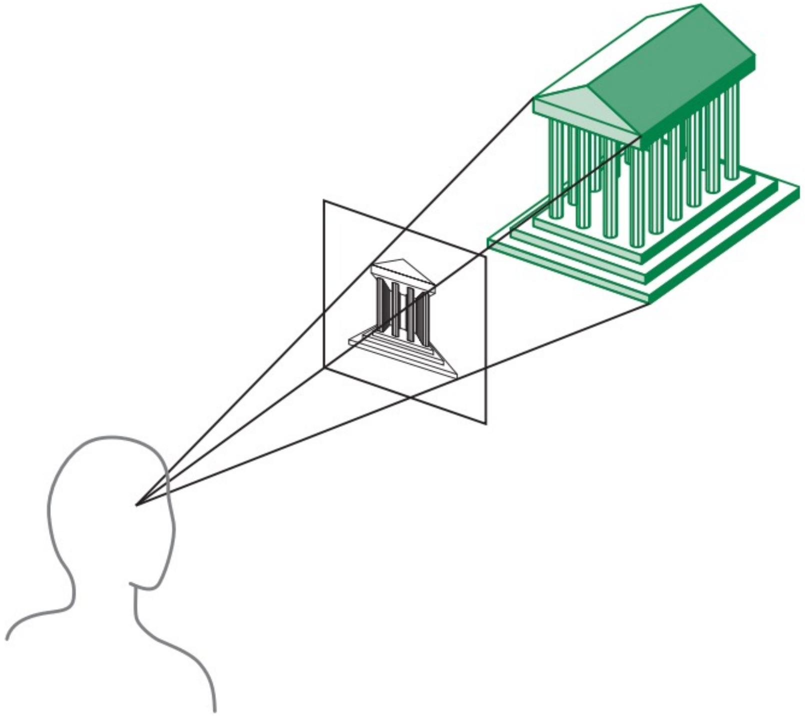
- All parallel lines remain parallel
- Objects don’t lose scale
  - close/far same size
- Useful for design renderings
- Not good when you want a scene to look natural

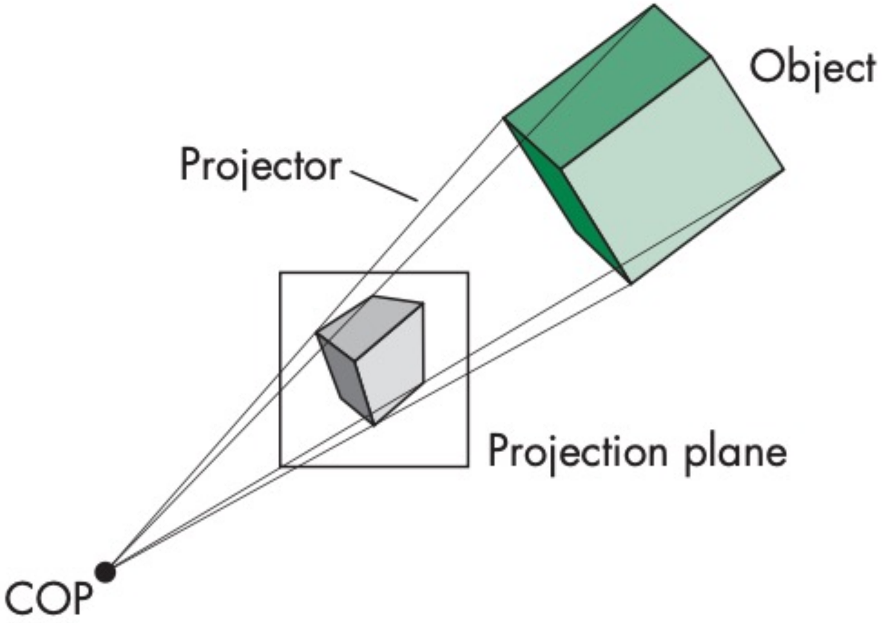


# Perspective Projection

# Perspective Viewing

- Projectors are no longer parallel (Orthographic Viewing)
- Instead, they converge on a Center of Projection (COP)





# The Concept

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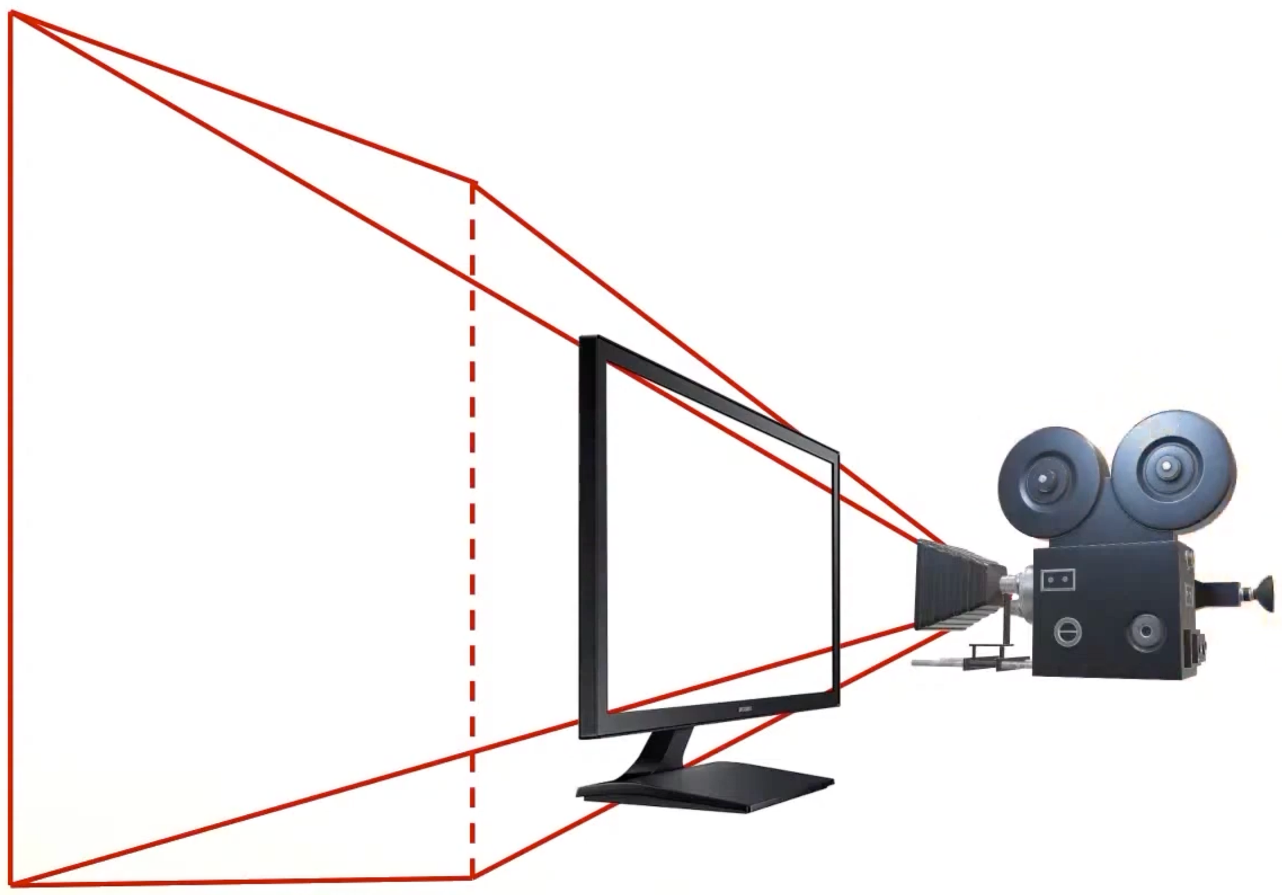
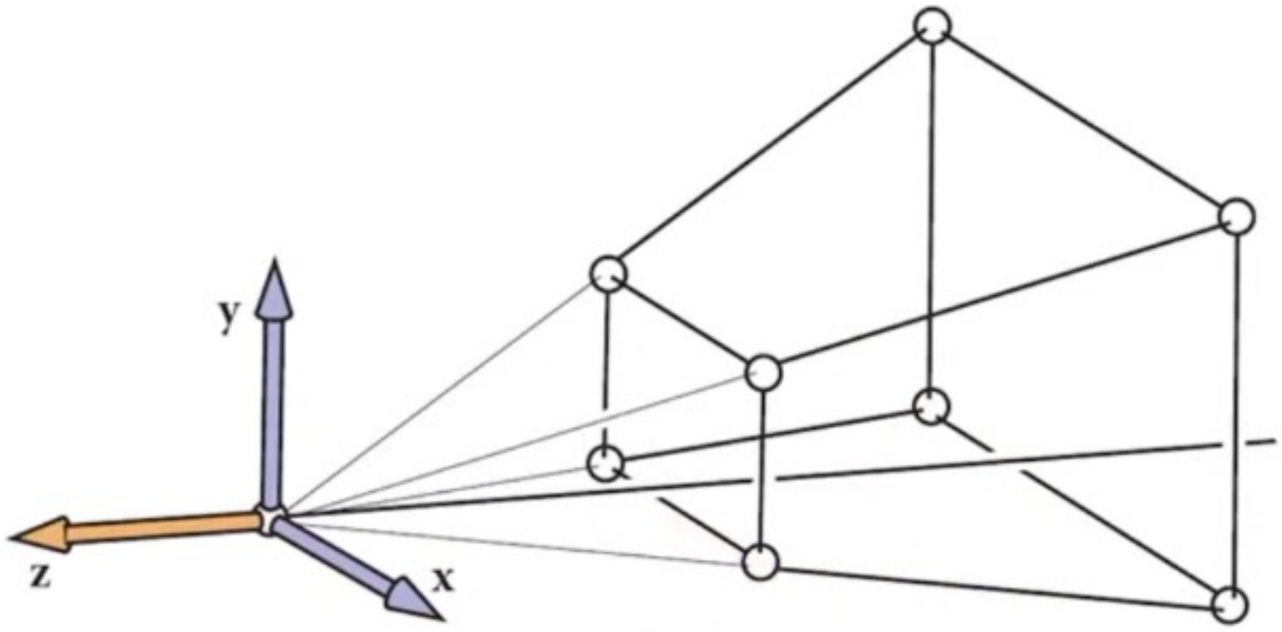


# A Different Viewing Volume



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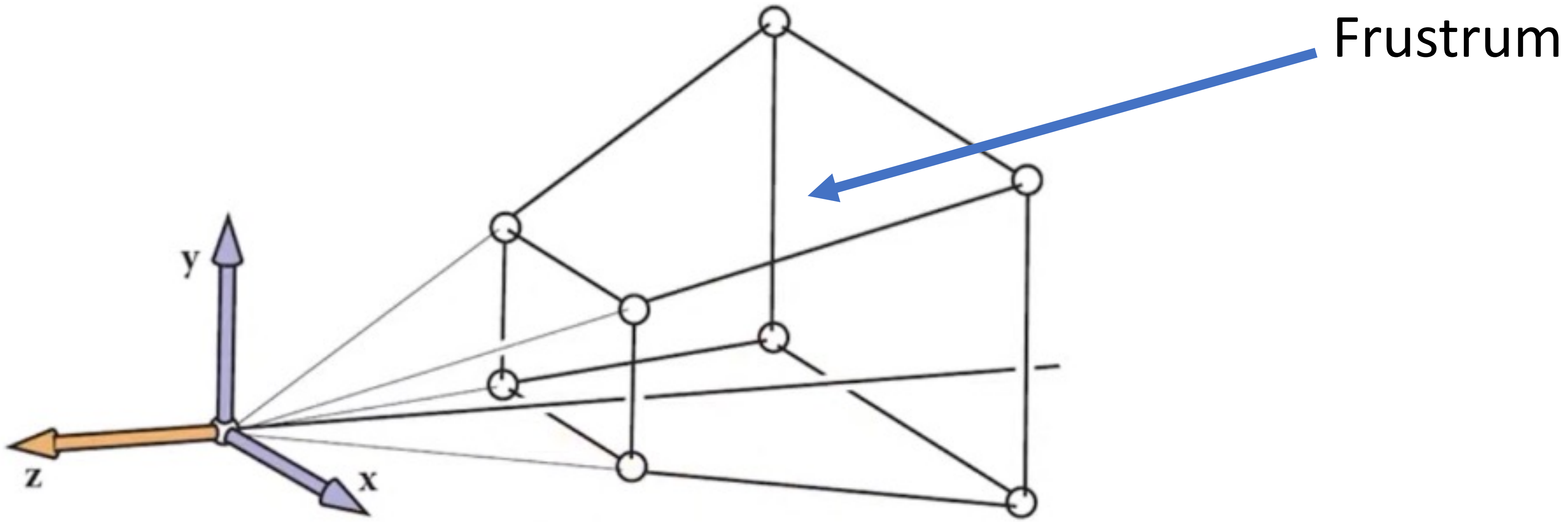

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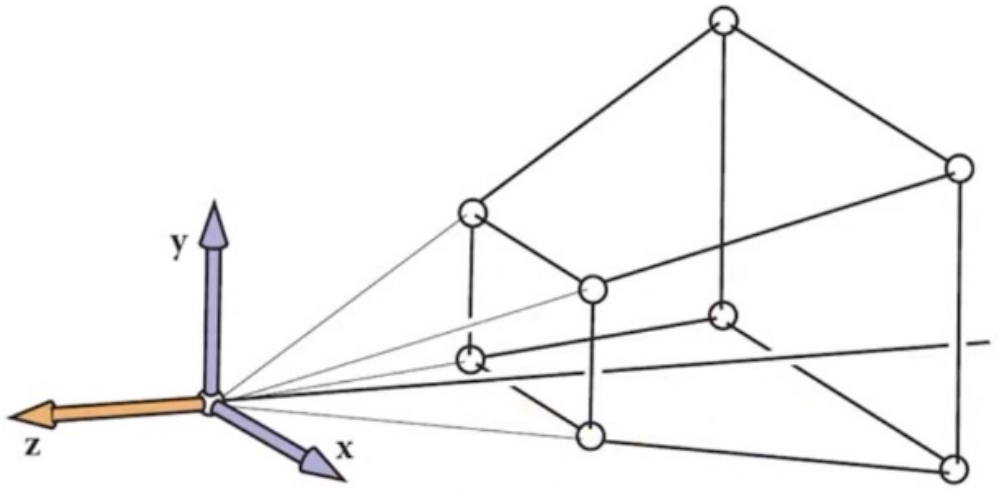


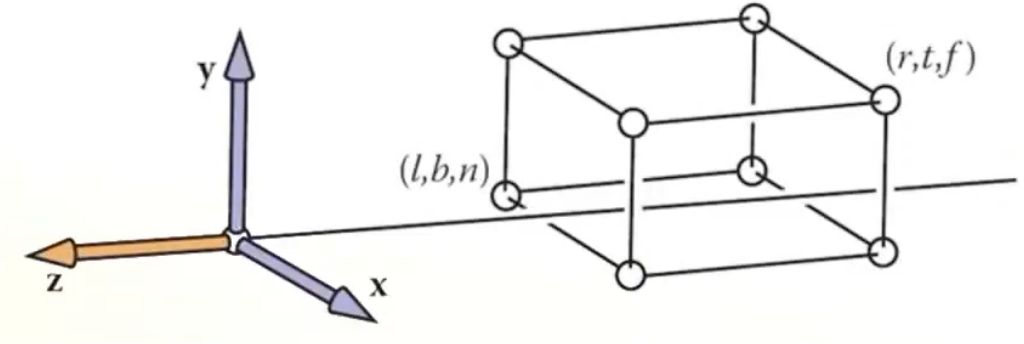

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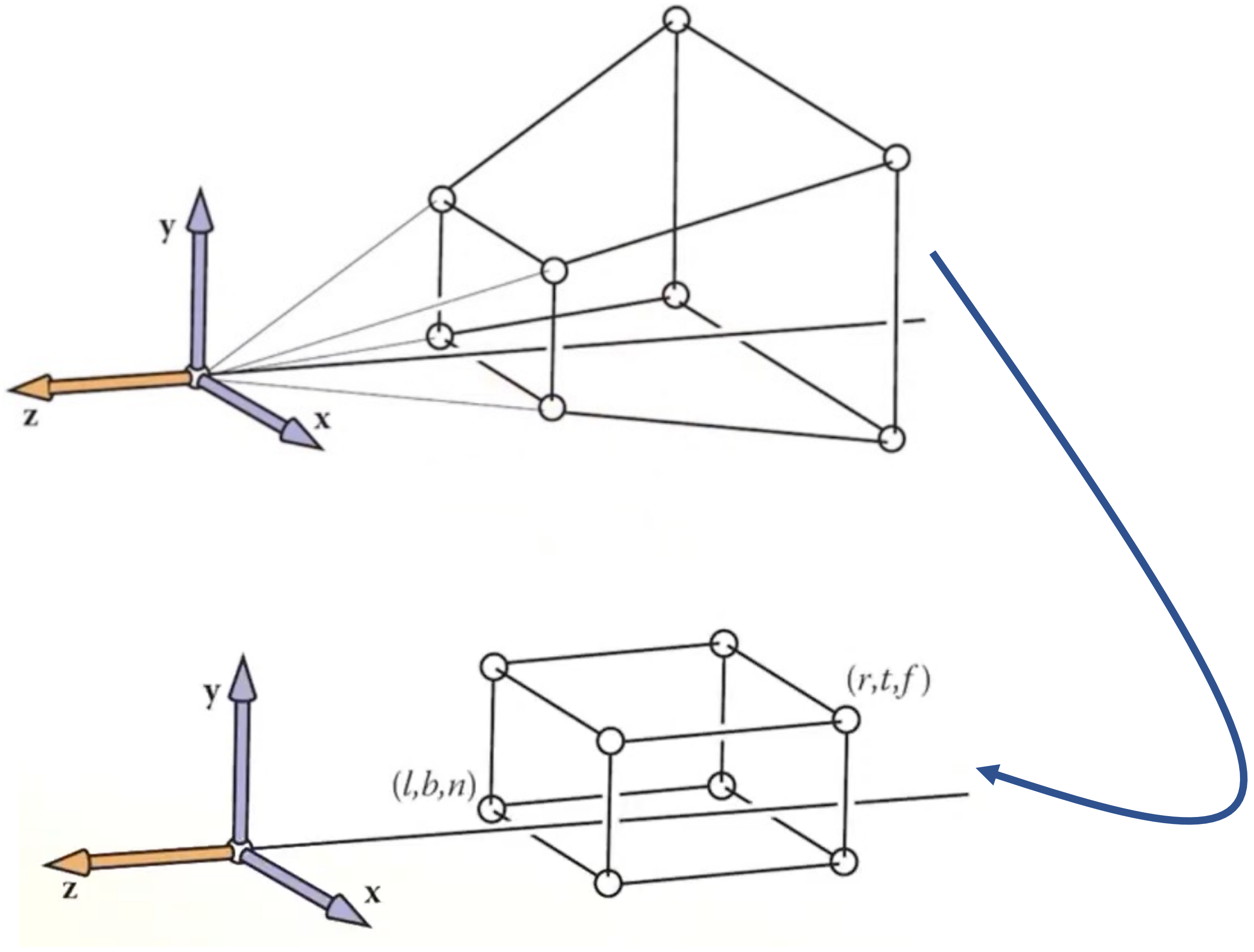
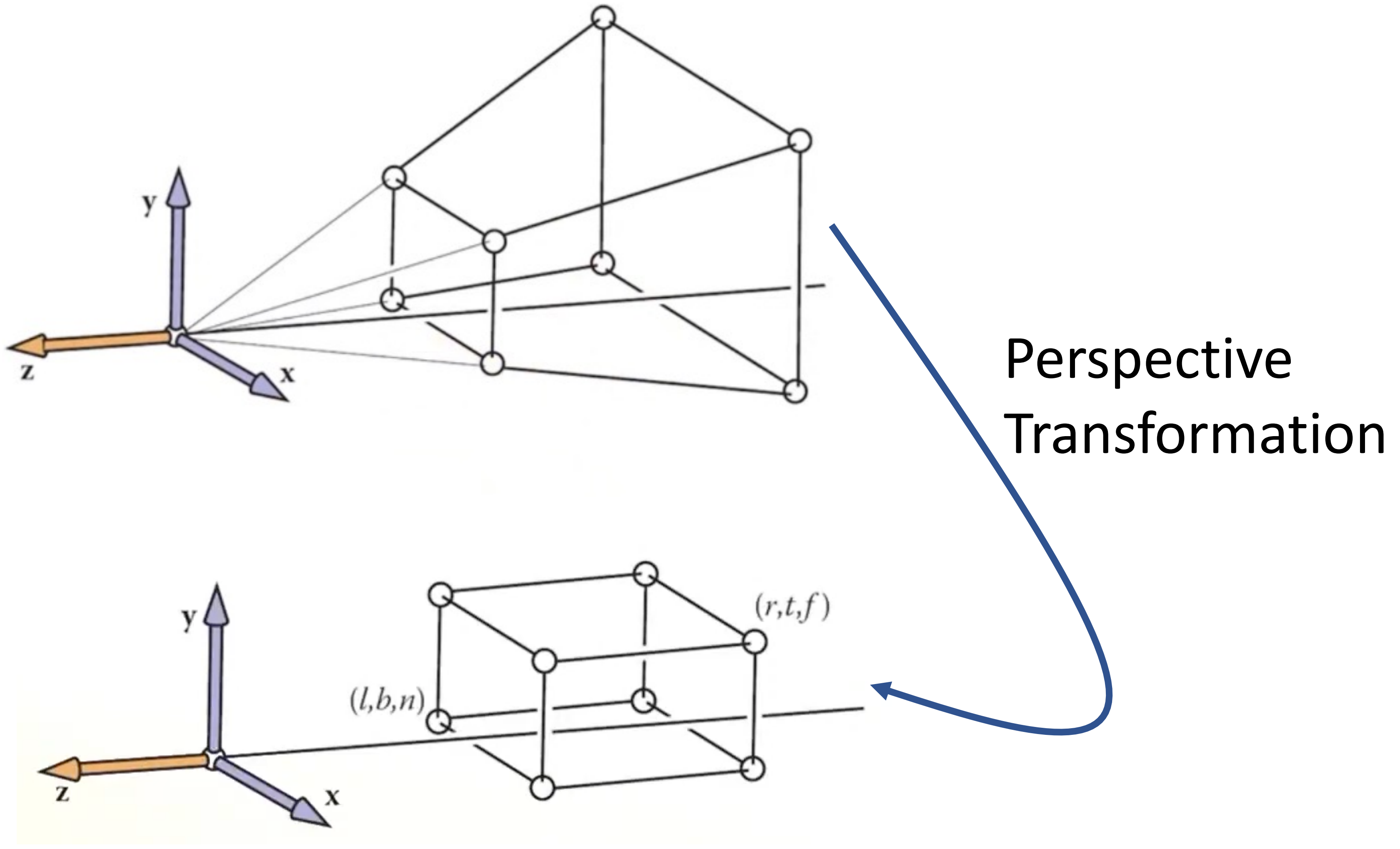


# Prospective Projection



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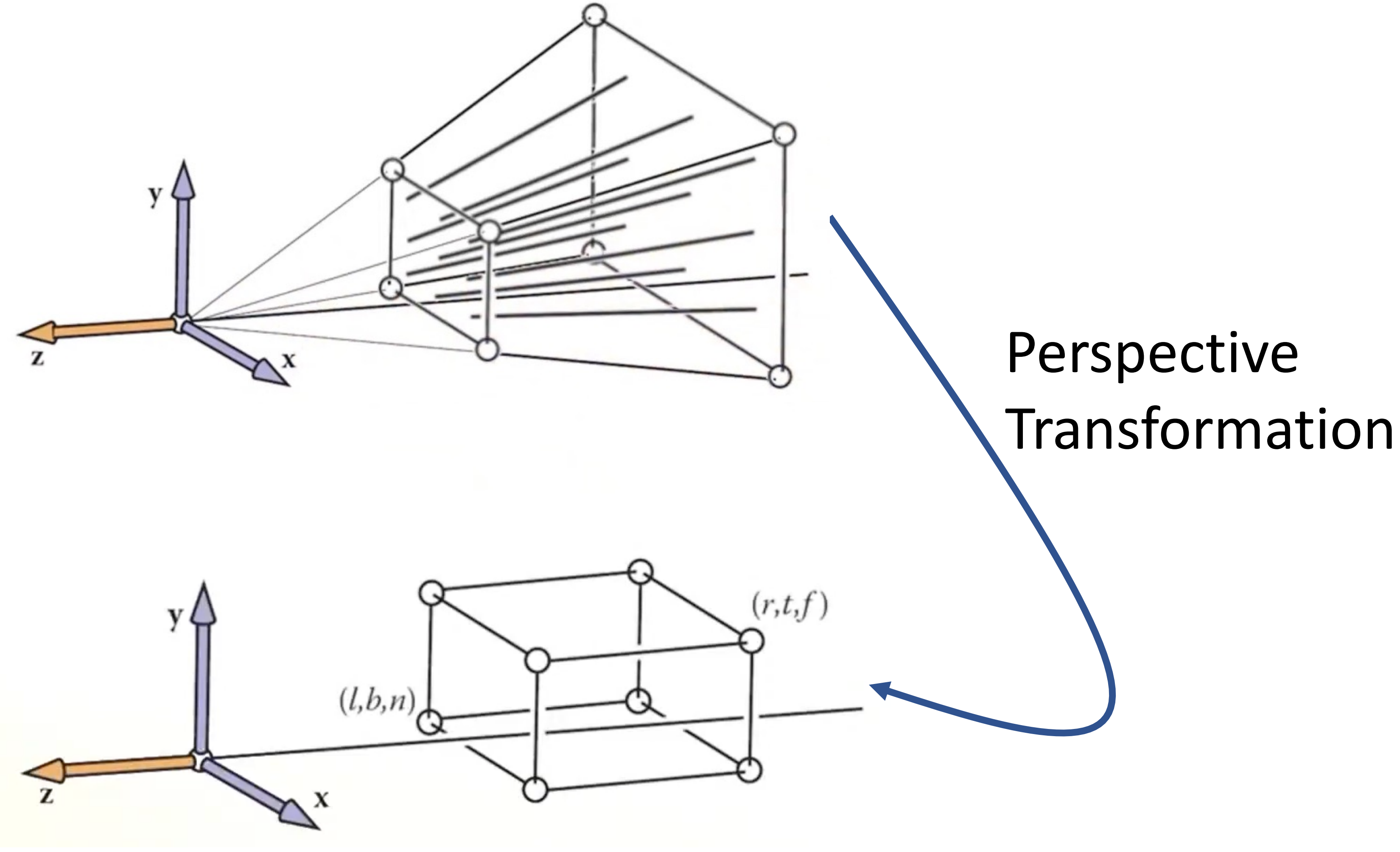

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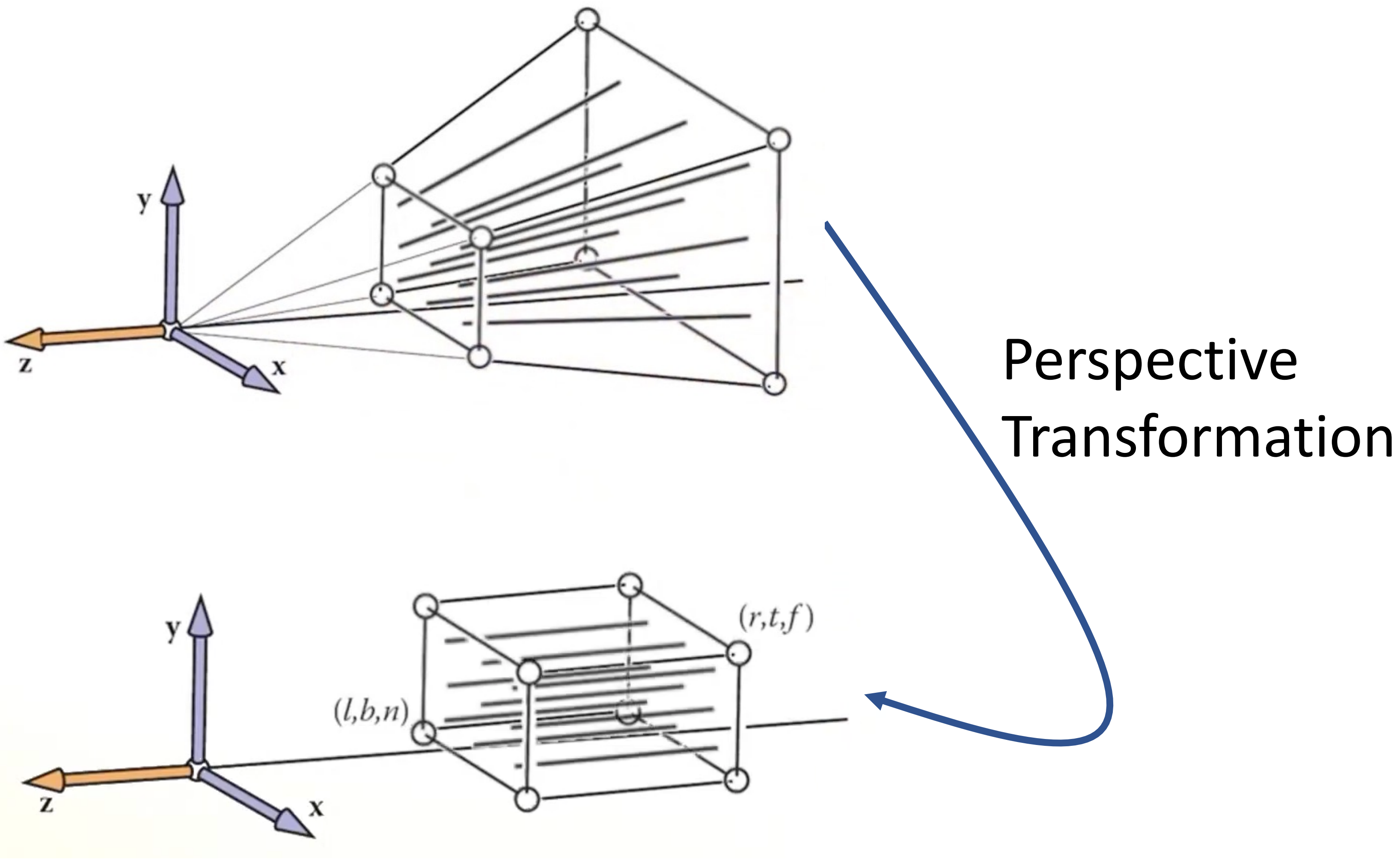
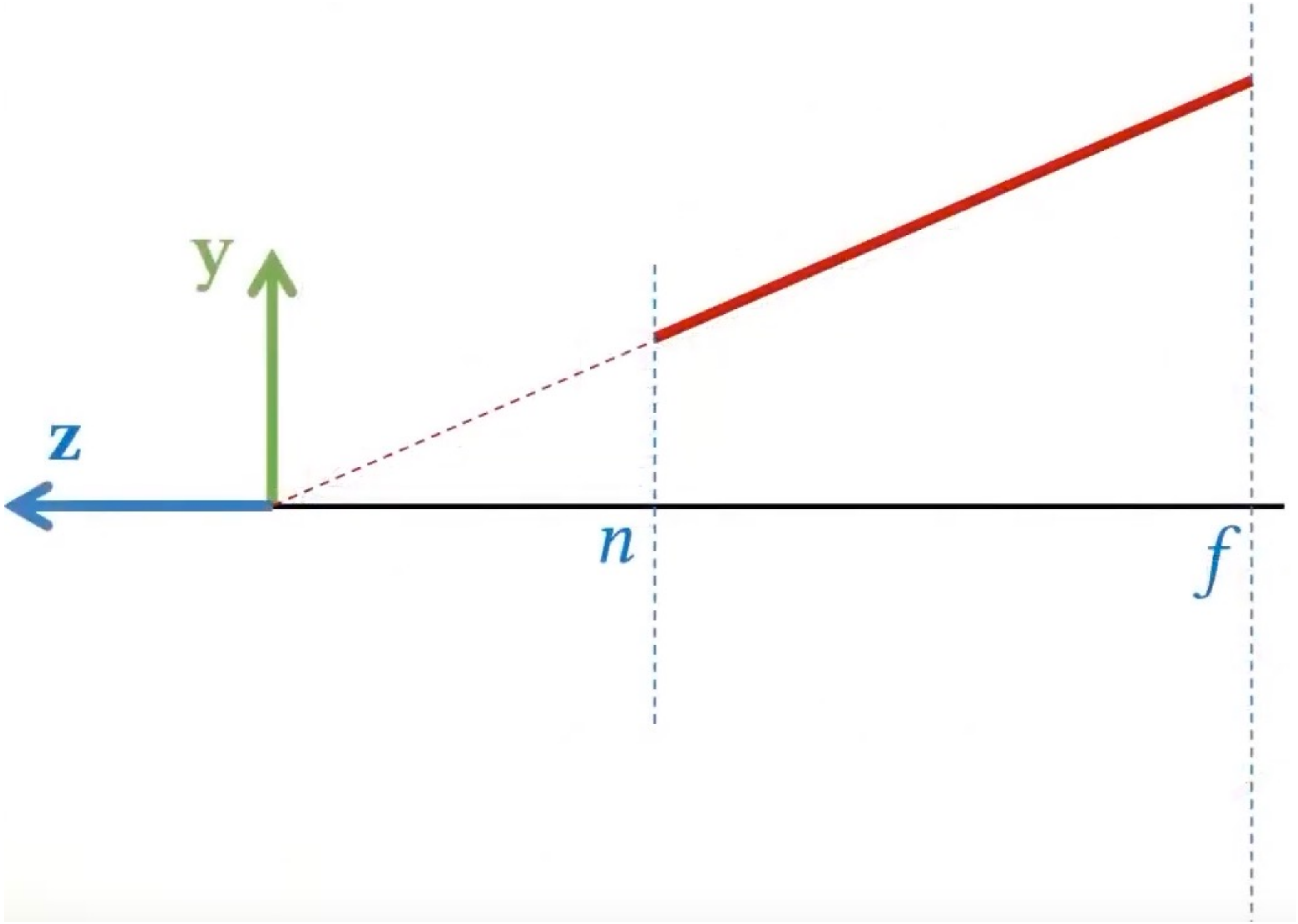


# Perspective Transformation



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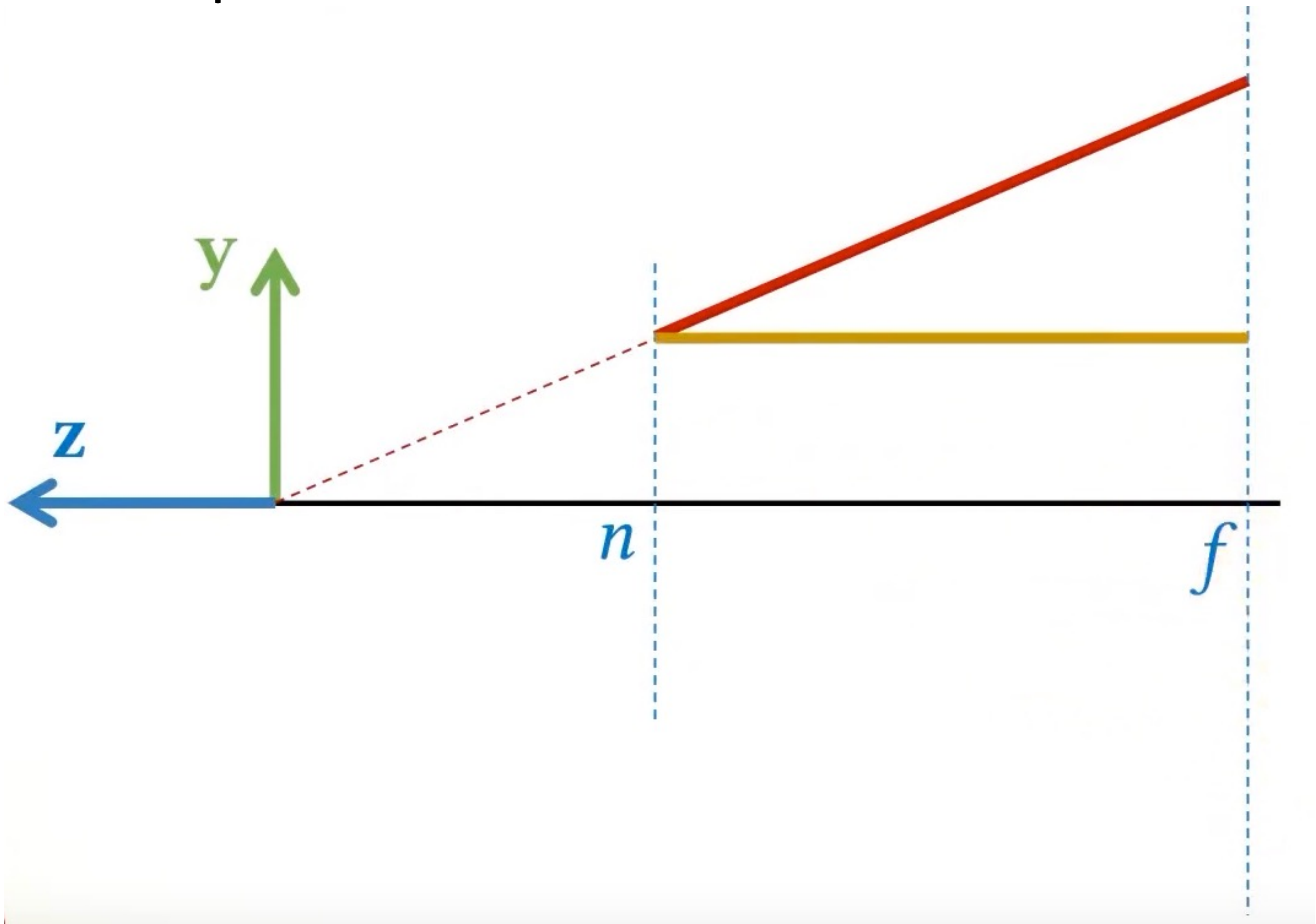

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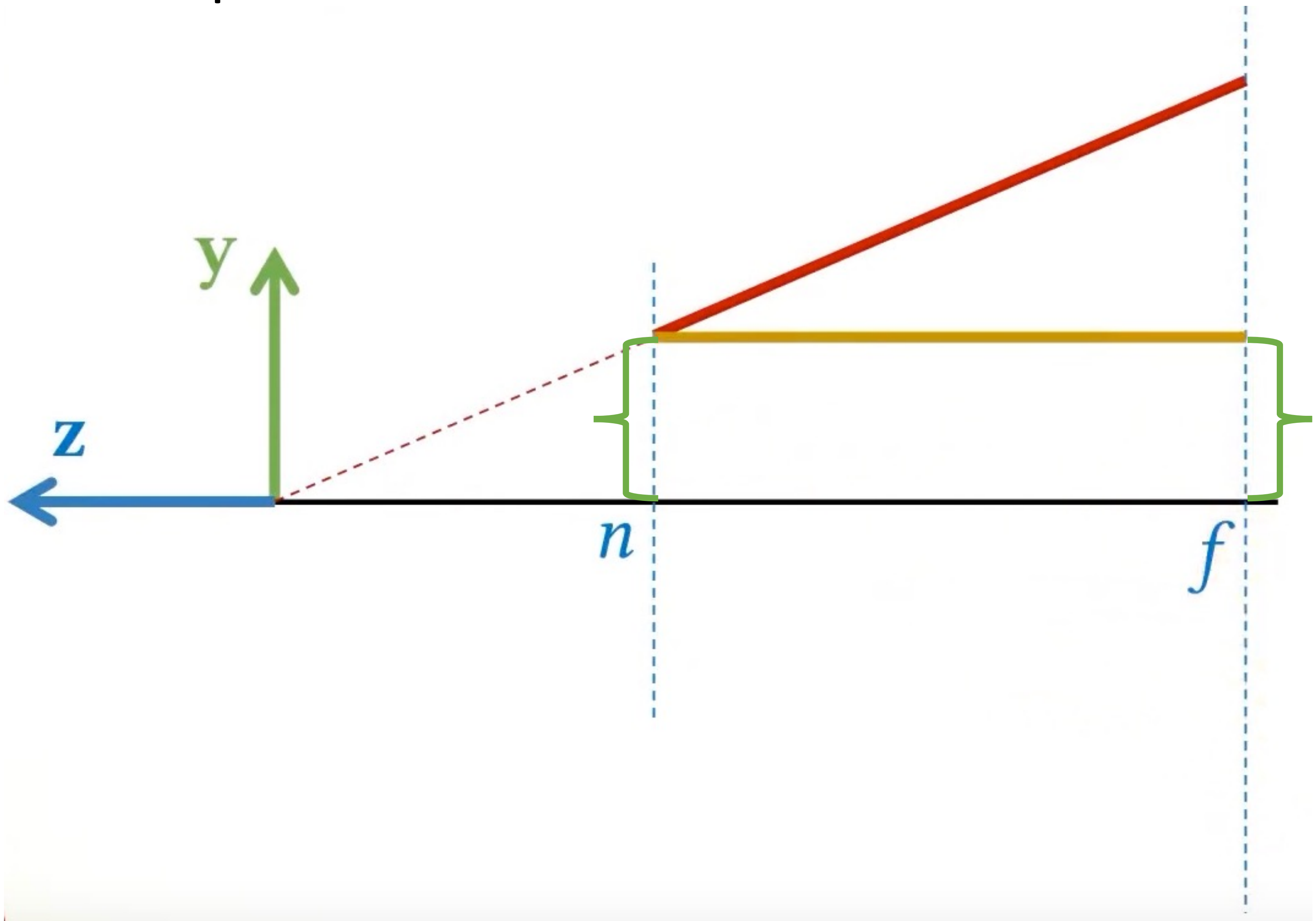

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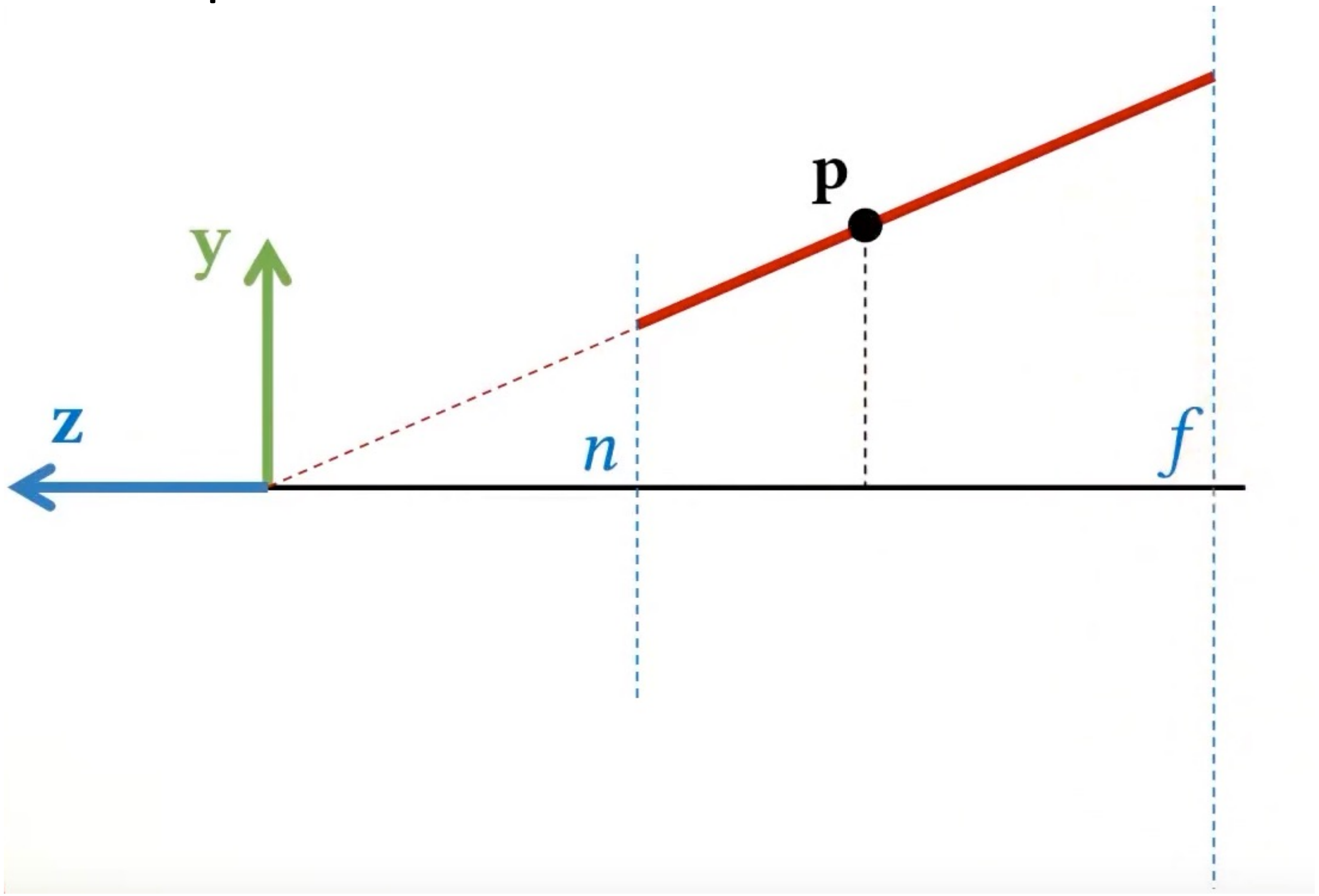

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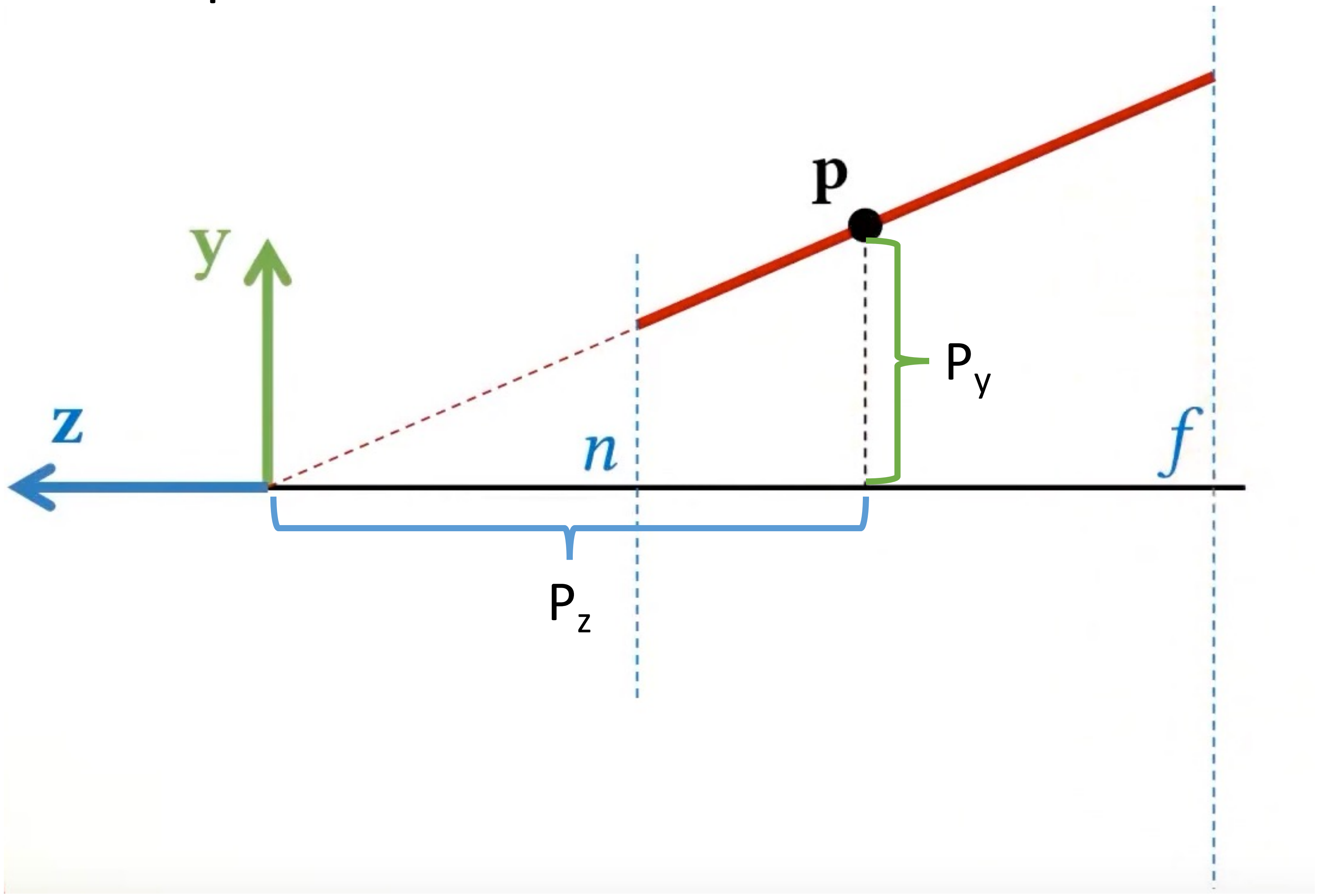

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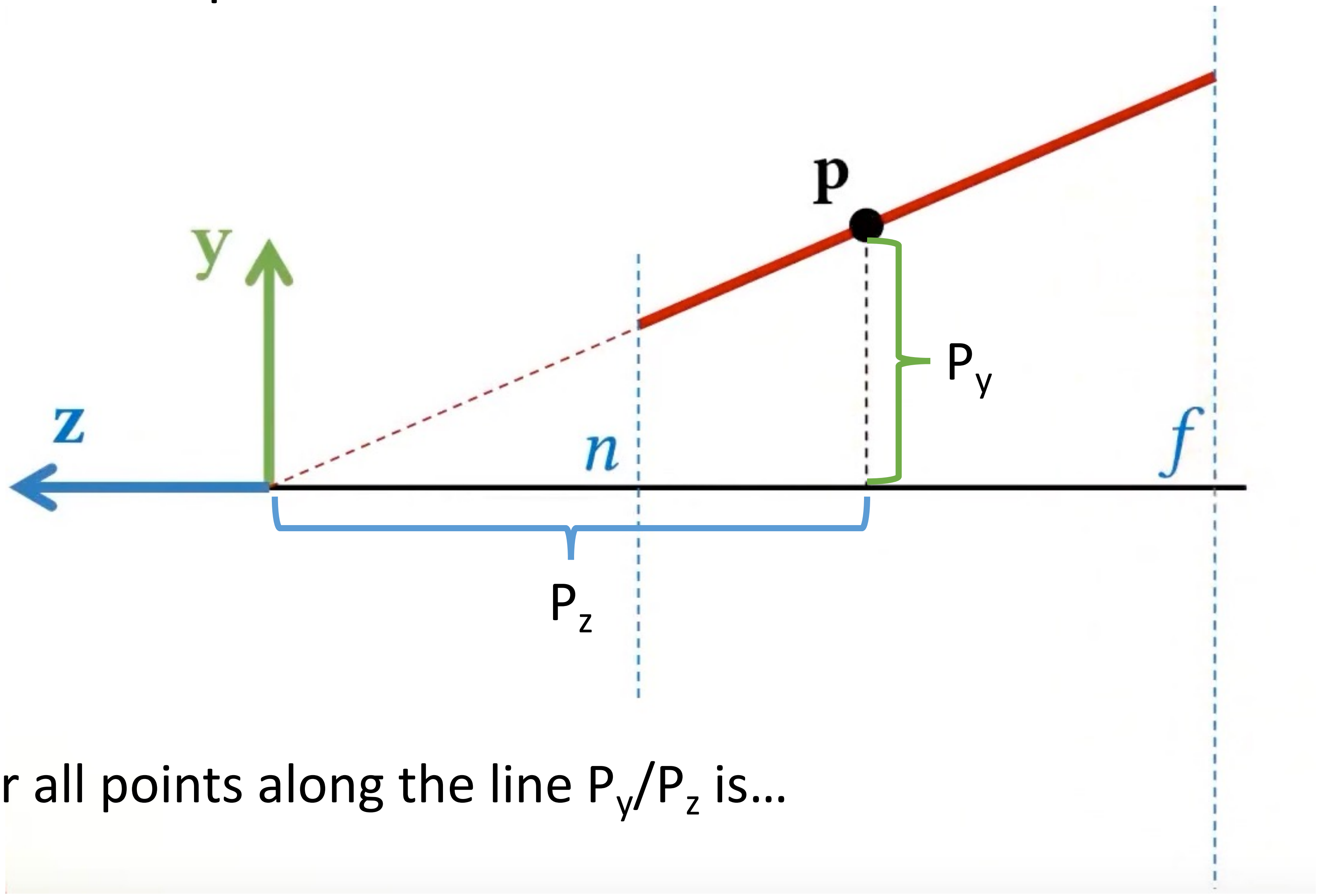
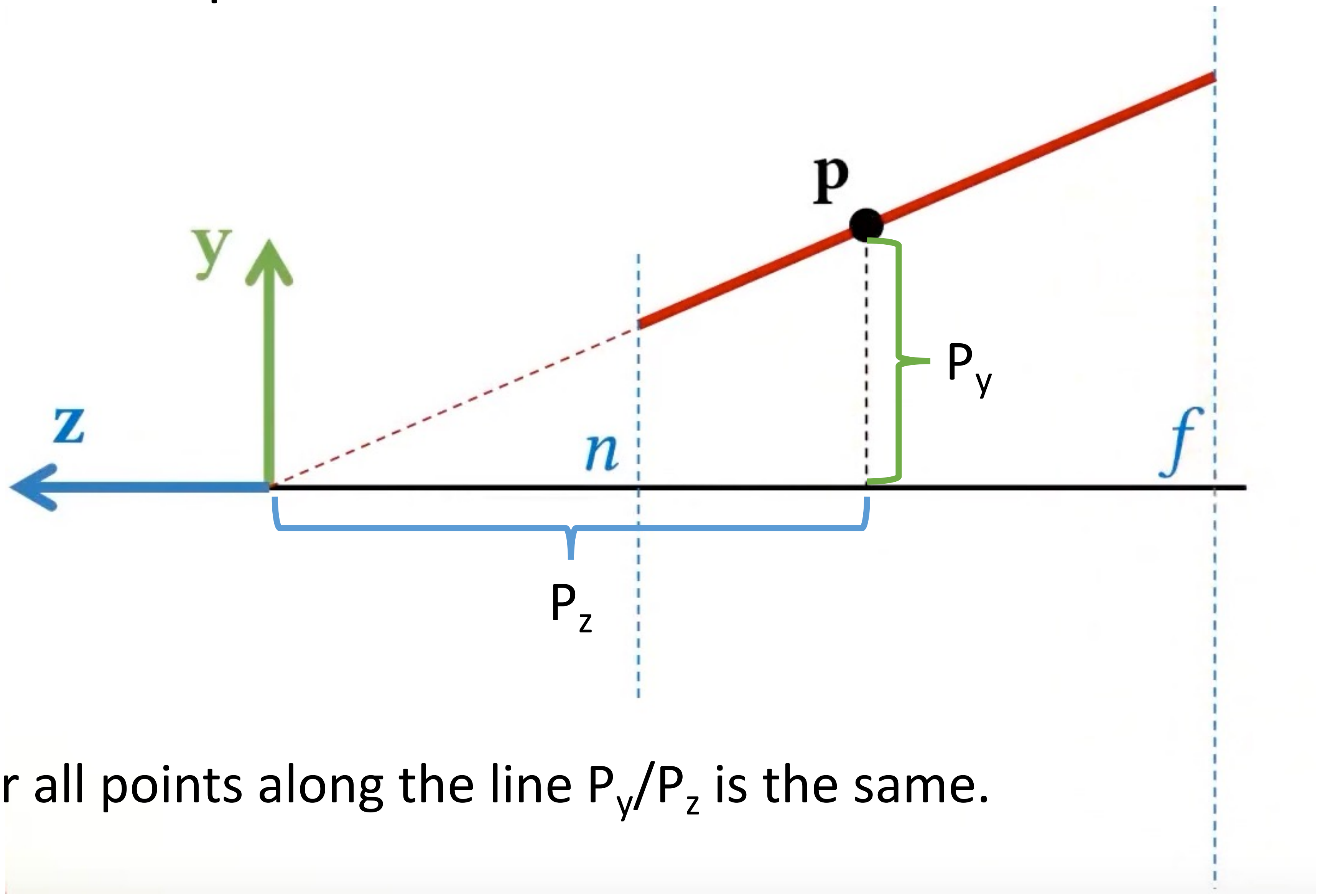


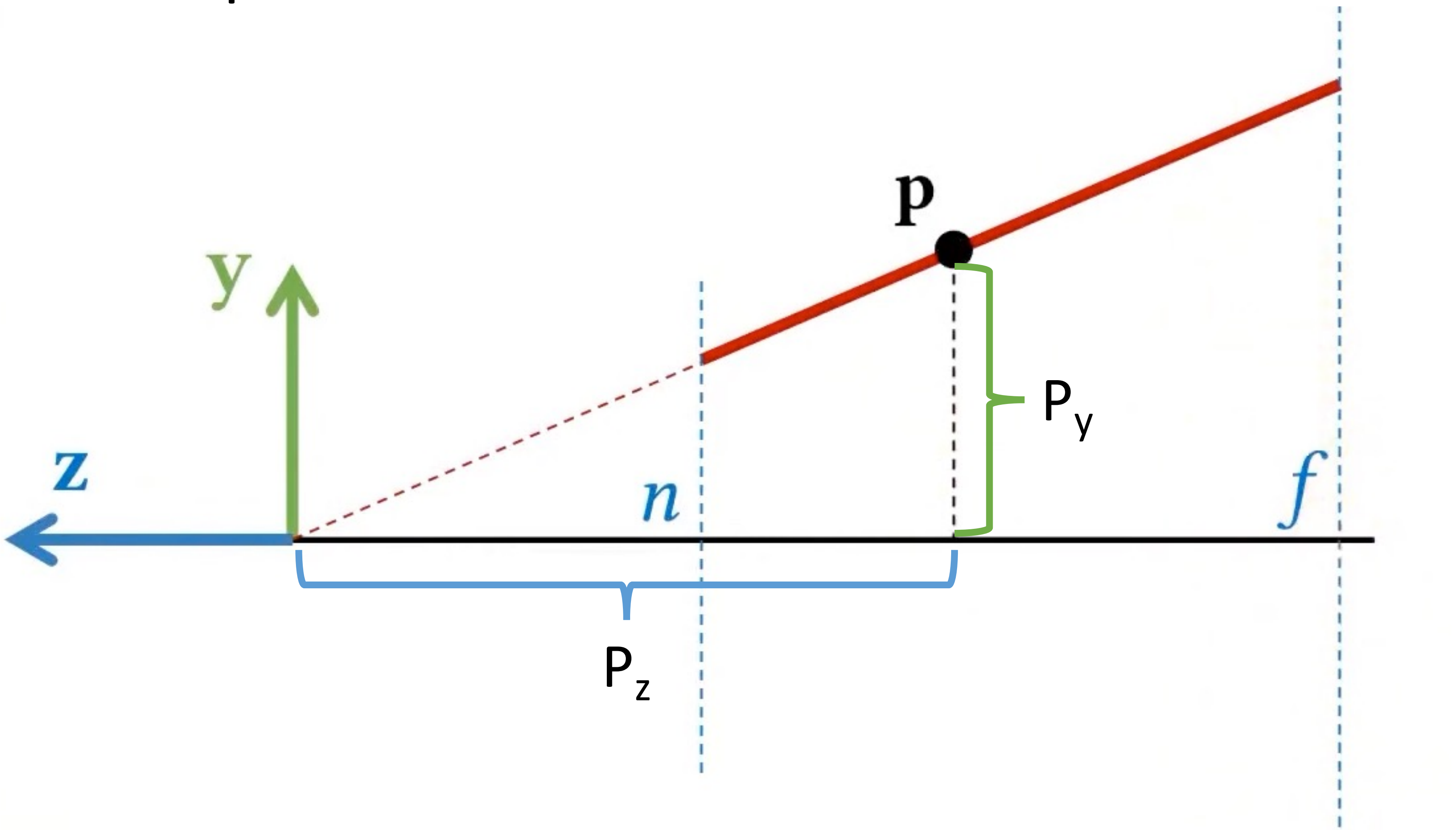

For all points along the line $P_y/P_z$ is…

# Perspective Transformation



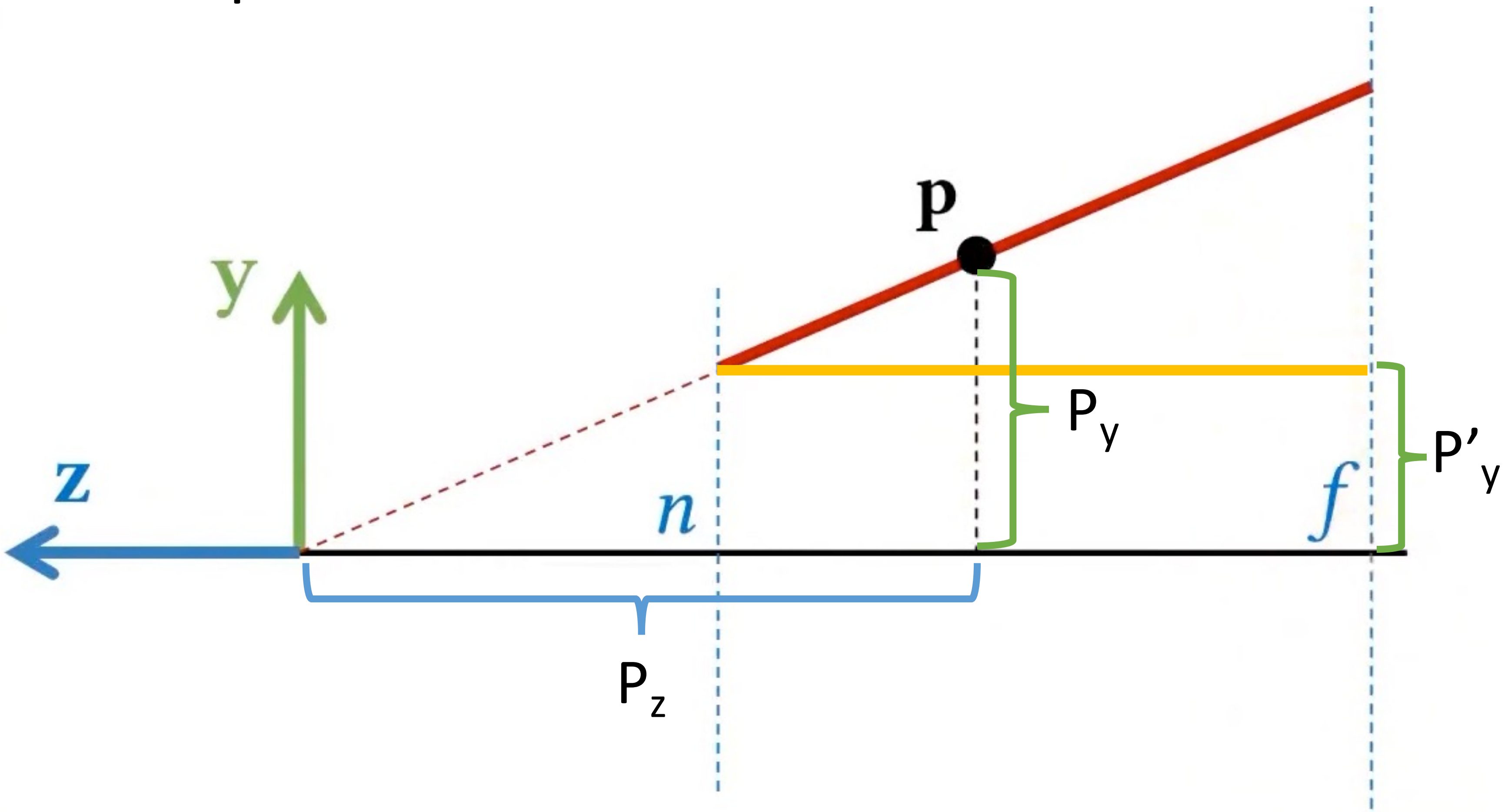

For all points along the line $P_y/P_z$ is the same.

# Perspective Transformation




For all points along the line $P_y/P_z$ is the same.
The tangent of the angle!

# Perspective Transformation

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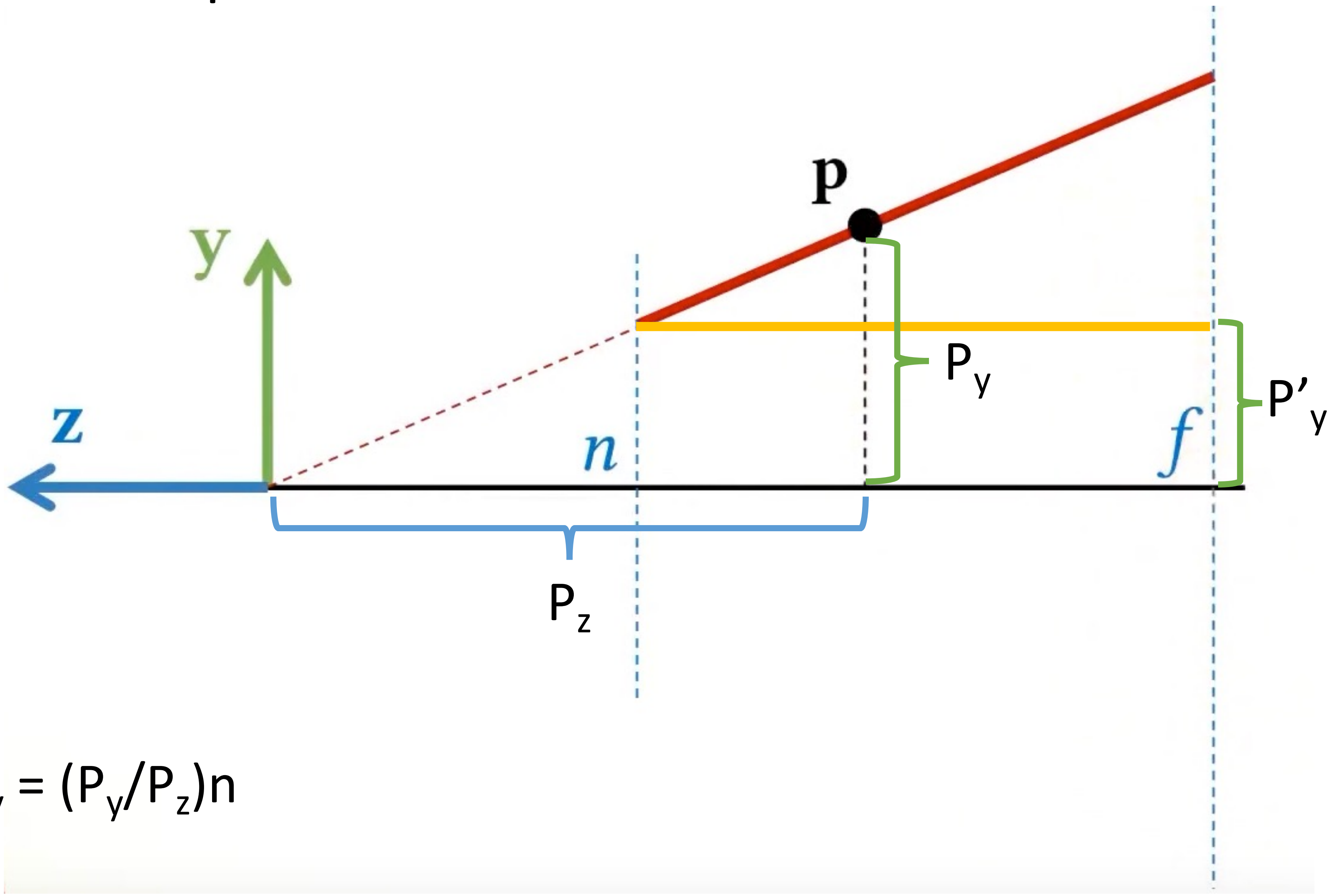
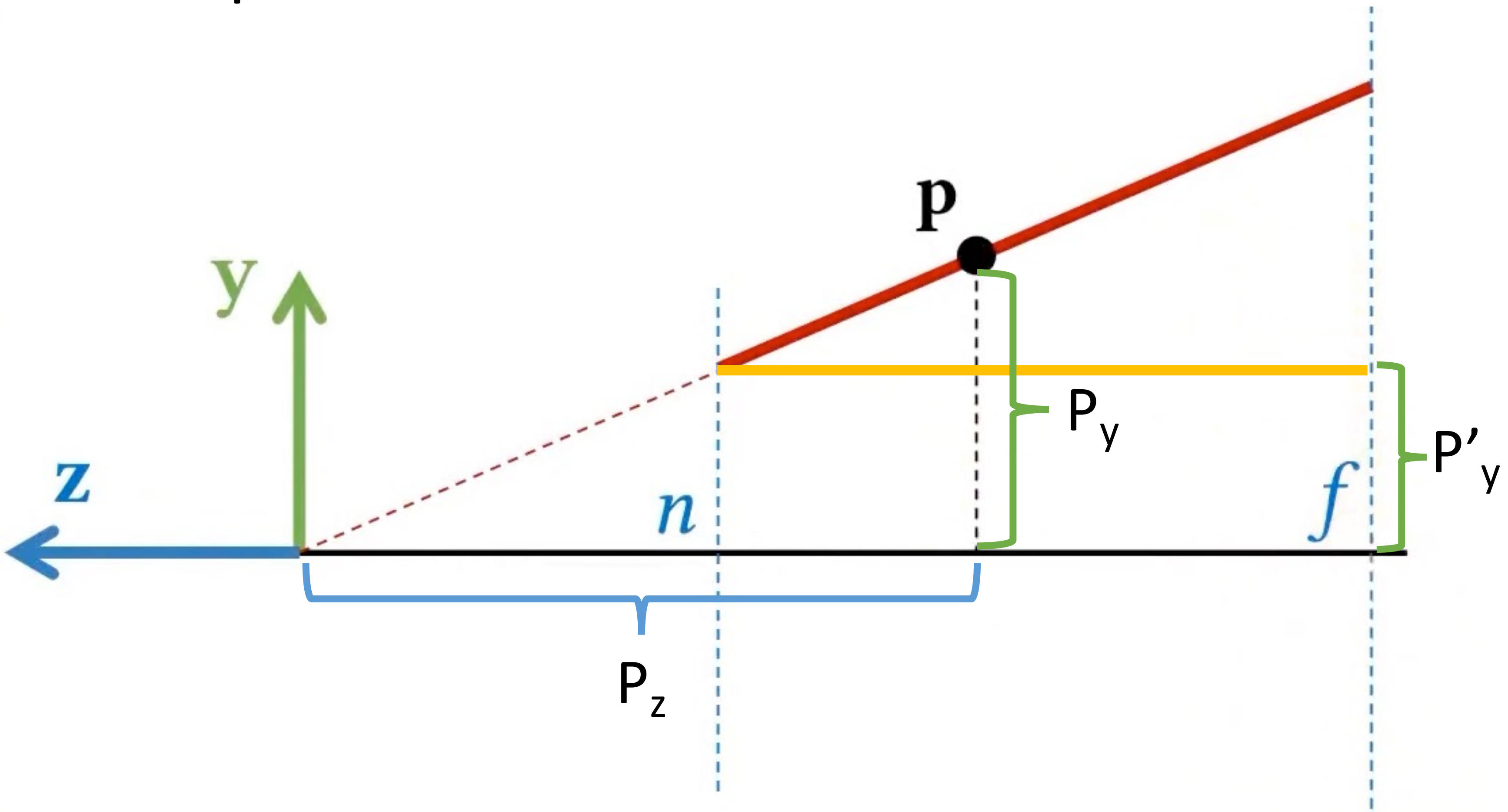



$P'_y = (P_y/P_z)n$

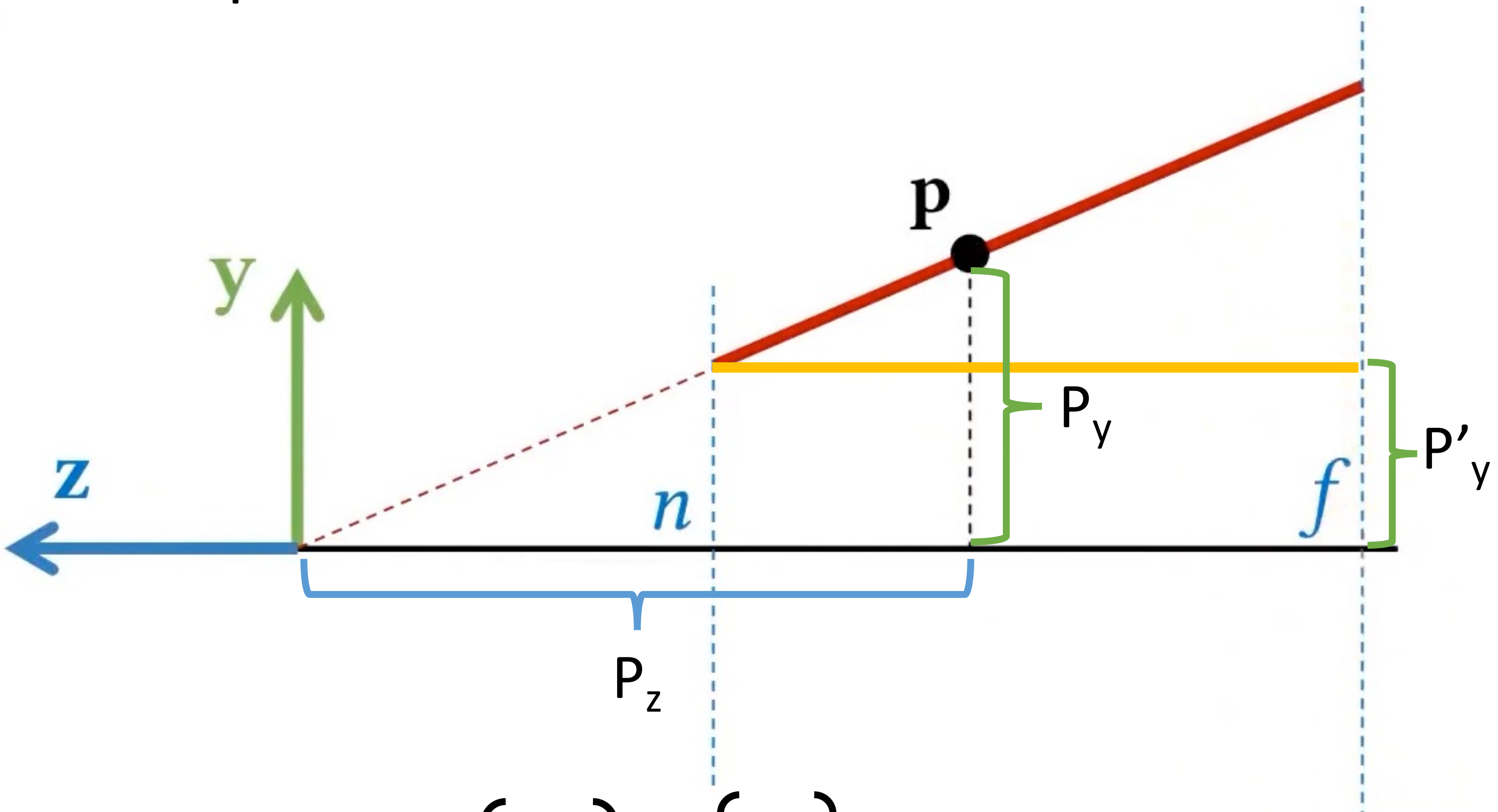
# Perspective Transformation




$P'_y = (P_y/P_z)n$

$P'_x = (P_x/P_z)n$

# Perspective Transformation




Written as a vector: $\begin{pmatrix} P'_x \\ P'_y \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \end{pmatrix} \frac{n}{P_z}$

# Homogeneous Coordinates

- Initially we said that the last component was going to be 1 for points and 0 for vectors

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}$$

# Homogeneous Coordinates

- Initially we said that the last component was going to be 1 for points and 0 for vectors
- Now we will extend the definition to use that component to help us with our perspective transformation

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$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \\ 1 \end{pmatrix} \equiv \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix}$$

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$P_z = n$

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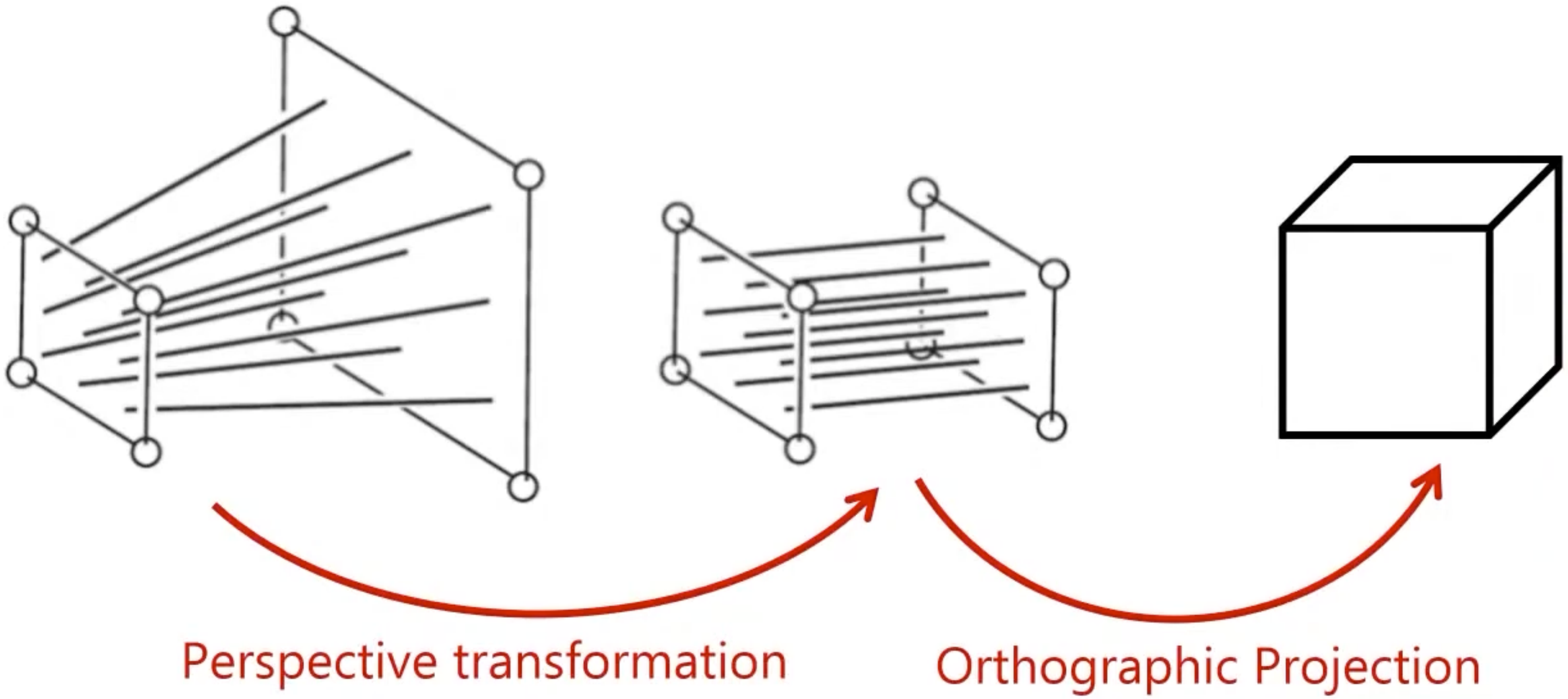
$P_z = f$ ⇒ $P'_z = (\,(n+f)P_z - fn) / P_z$

$= (n+f) - fn / P_z$

$= (n+f) - n$

$= f$

# Perspective Projection




$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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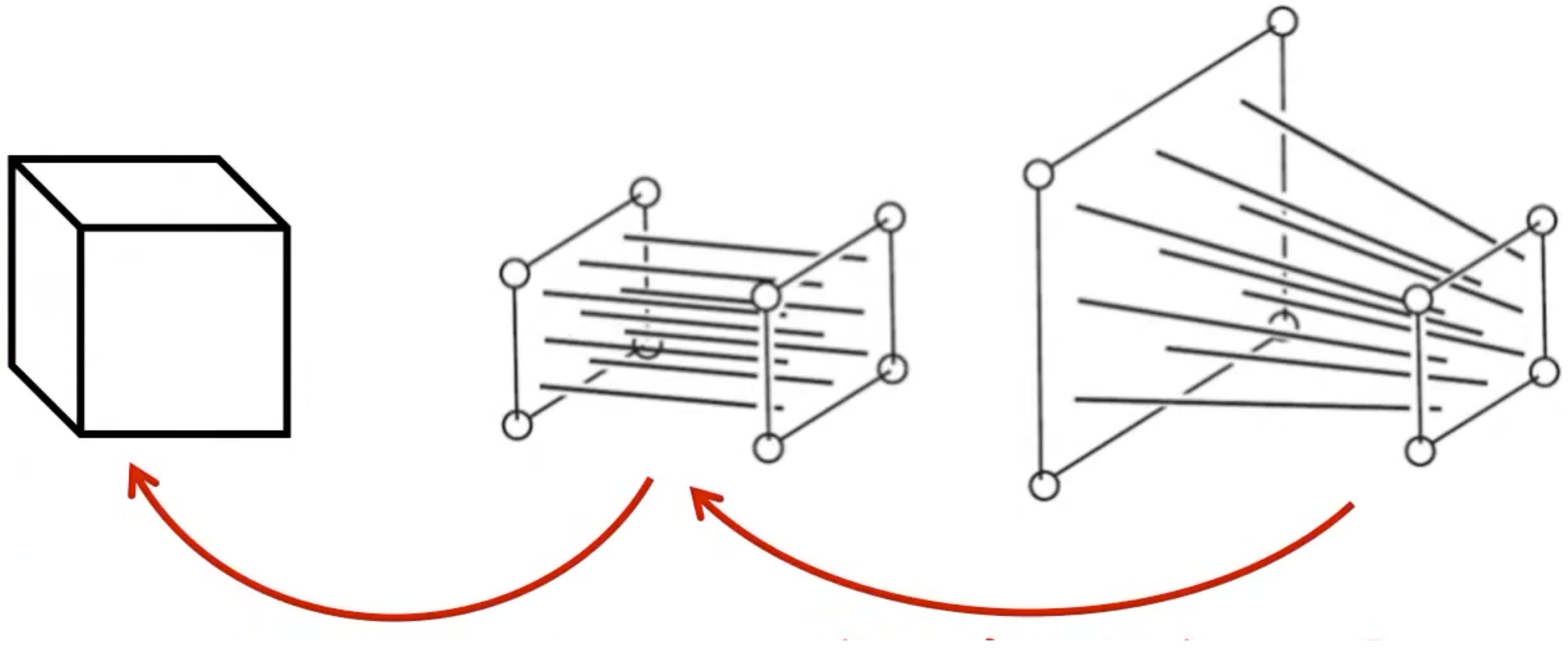

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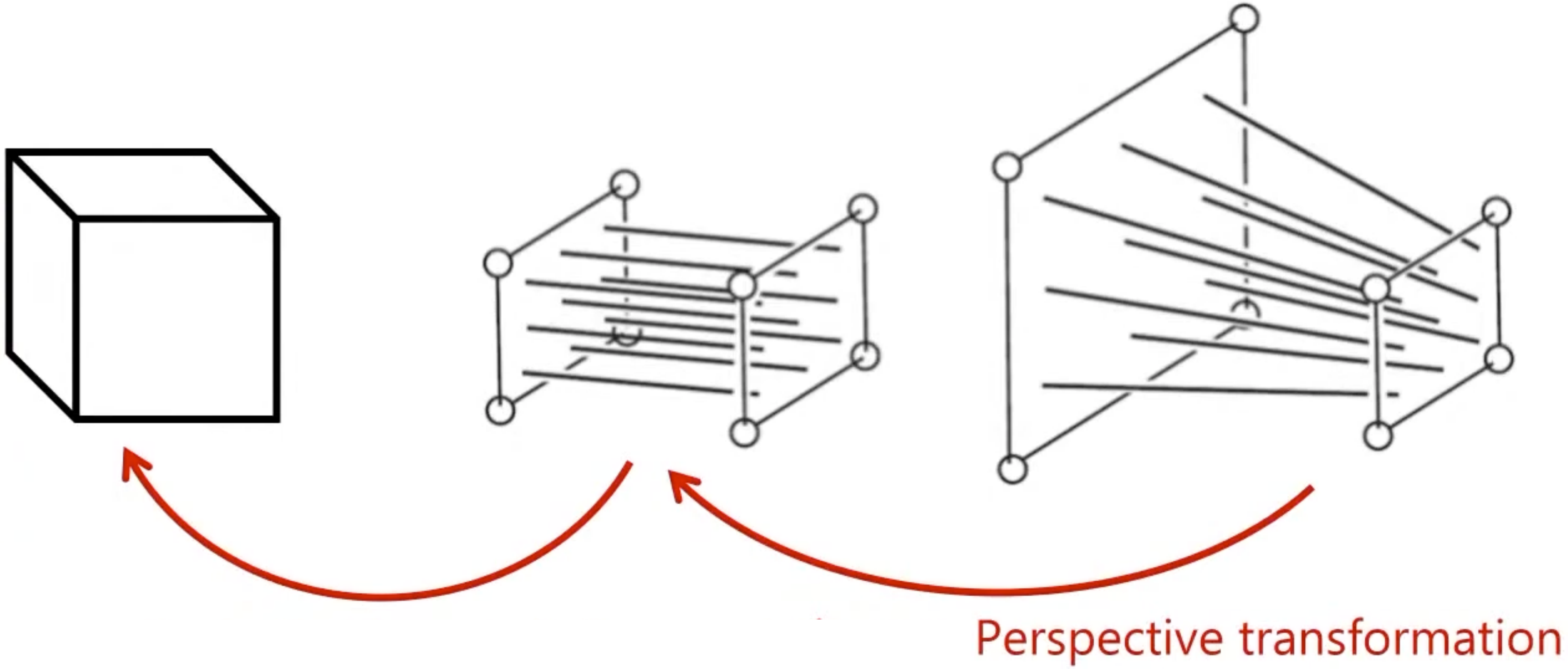


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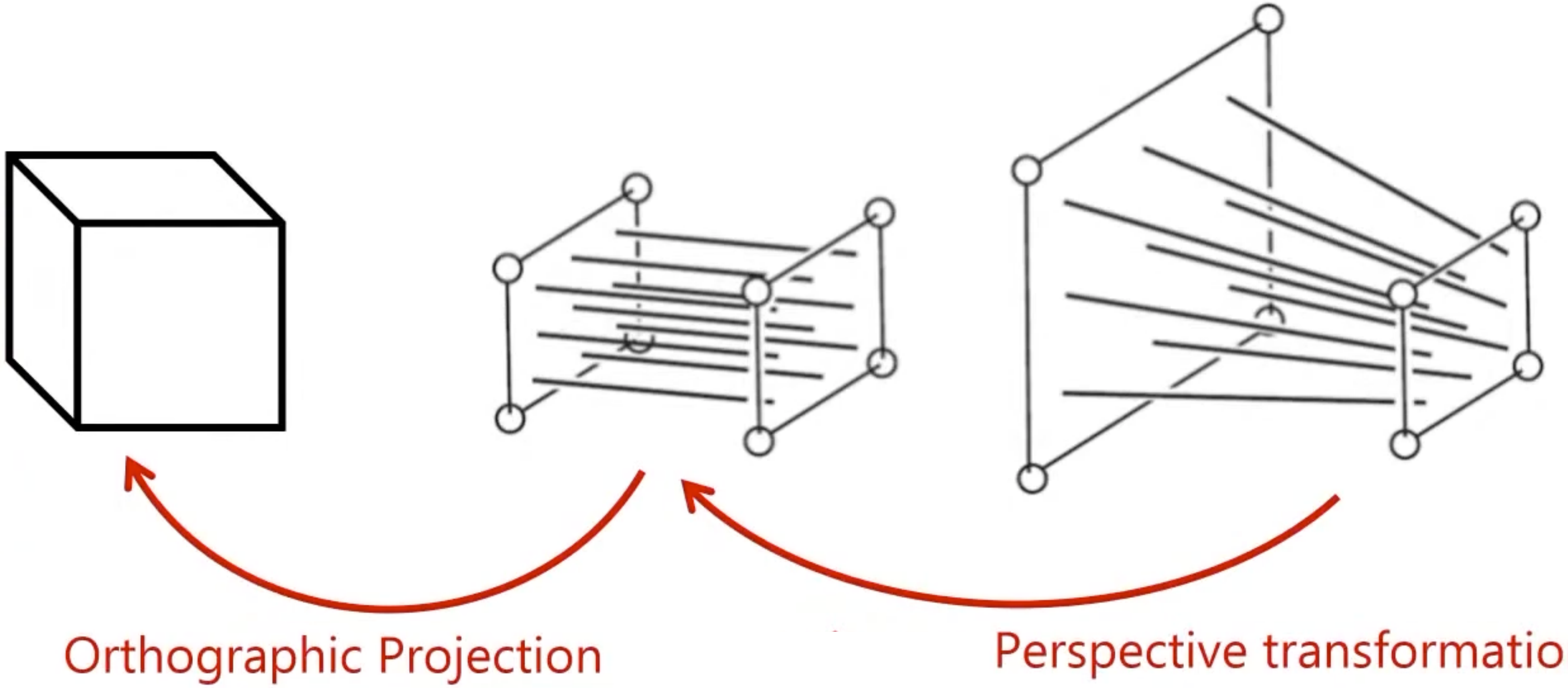


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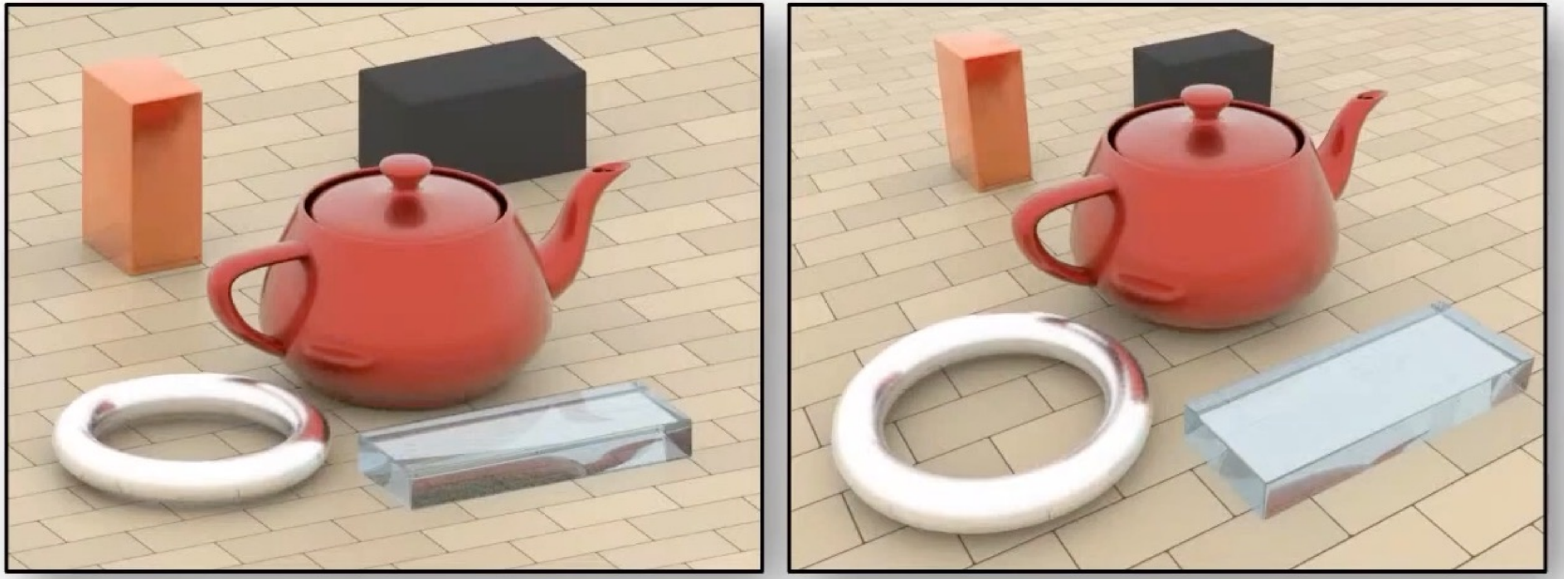

**Orthographic** projection

**Perspective** projection