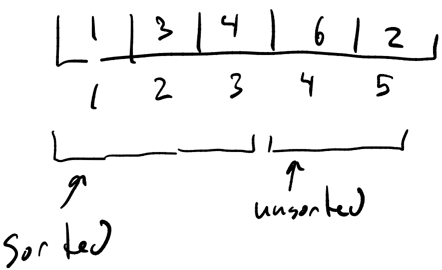
INSERTION-SORT(A, n)

for $j = 2$ **to** n
  $key = A[j]$
  // Insert $A[j]$ into the sorted sequence $A[1 \,..\, j-1]$.
  $i = j - 1$
  while $i > 0$ and $A[i] > key$
    $A[i+1] = A[i]$
    $i = i - 1$
  $A[i+1] = key$

Loop invariant

- True before first iteration
- It remains true before the next iteration
- When the loop terminates, the invariant and the termination condition help show correctness



j	Key	i
3	3	2

Invariant - At the start of each iteration, the subarray $A[1 \,..\, j-1]$ consists of the elements originally in $A[1 \,..\, j-1]$, but in sorted order

Outer loop has n-1 iterations

Inner loop

1 iteration first

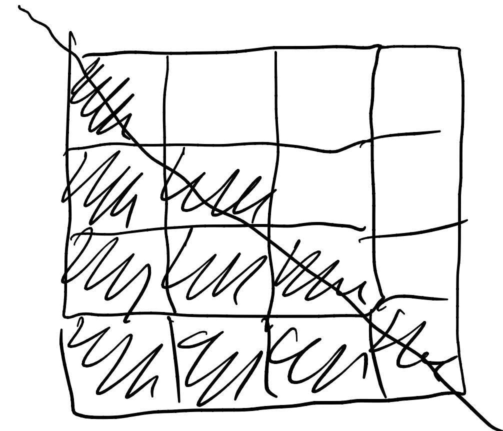
then 2

then 3

⋮

then n-1

In the worst case



$$1 + 2 + 3 + \cdots + n$$
$$n + n-1 + n-2 + \cdots + 1$$
$$\overline{(n+1) + (n+1) + (n+1) + \cdots + (n+1)}$$

$$\frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

Best case:

Inner loop's condition is checked, but it never iterates

(the list is already sorted)

INSERTION-SORT(A, n)	*cost*	*times*
for $j = 2$ **to** n	c_1	n
$\quad key = A[j]$	c_2	$n - 1$
$\quad$ // Insert $A[j]$ into the sorted sequence $A[1 \mathinner{..} j - 1]$.	0	$n - 1$
$\quad i = j - 1$	c_4	$n - 1$
$\quad$ **while** $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
$\qquad A[i + 1] = A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$\qquad i = i - 1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$\quad A[i + 1] = key$	c_8	$n - 1$

$\Theta(g(n)) = \{ f(n) :$ there exist positive constants c_1, c_2, and n_0
such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0 \}$

$O(g(n)) = \{ f(n) :$ there exist positive constants c and n_0
such that $0 \le f(n) \le c\, g(n)$ for all $n \ge n_0 \}$

$\Omega(g(n)) = \{ f(n) :$ there exist positive constants c and n_0
such that $0 \le c\, g(n) \le f(n)$ for all $n \ge n_0 \}$

Insertion sort in the worst case is

$$\underbrace{O(n^2), \; \Theta(n^2), \; \Omega(n^2), \; O(n^3), \; \Omega(n), \; \ldots}$$

not useful information