

HW #5: Design techniques - greedy method, divide and conquer, dynamic programming

Directions: Complete your work on a separate sheet of paper. Submit the physical copy of your work at the beginning of class on the specified due date. Show your work. You may work in groups of up to 3 students provided that all students participate in each question. Provide a short preliminary explanation of how an algorithm works before running an algorithm or presenting a formal algorithm description, and use examples or diagrams if they are needed to make your presentation clear.

1. For each of the following recurrence equations which describe the running time $T(n)$ of a recursive algorithm, use the master method to express the asymptotic complexity (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$).
 - (a) $T(n) = 2T(n/2) + \log n$
 - (b) $T(n) = 8T(n/2) + n^2$
 - (c) $T(n) = 7T(n/3) + n$
 - (d) $T(n) = 4T(n/2) + n^2$
 - (e) $T(n) = 3T(n/2) + n^2$

2. For each of the following recurrence equations which describe the running time $T(n)$ of a recursive algorithm, use the master method to express the asymptotic complexity (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$).
 - (a) $T(n) = T(7n/10) + n$
 - (b) $T(n) = 16T(n/4) + n^2$
 - (c) $T(n) = 4T(n/3) + n \log n$
 - (d) $T(n) = 4T(n/2) + n^2 \sqrt{n}$
 - (e) $T(n) = 2T(n/2) + n^4$

3. Suppose we are given a set of activities specified by pairs of start times and finish times as $T = \{(5, 6), (9, 11)(3, 7), (1, 2), (10, 12), (6, 8), (1, 3), (7, 9), (1, 4), (11, 14), (2, 5), (4, 9), (7, 10)\}$. Solve the activity scheduling problem for these tasks.

4. Let $S = \{a, b, c, d, e, f, g\}$ be a collection of items with weight-benefit values as follows: $a(3, \$12)$, $b(6, \$12)$, $c(6, \$9)$, $d(1, \$5)$, $e(2, \$5)$, $f(10, \$10)$, $g(3, \$9)$. For example, item a weighs 3 lbs and is worth a total of \$12. Assume you have a knapsack that can hold a total of 11 lbs. What is an optimal solution to the **0-1** knapsack problem for S ? Show your work.

5. Let $S = \{a, b, c, d, e, f, g\}$ be a collection of items with weight-benefit values as follows: $a(3, \$12)$, $b(6, \$12)$, $c(6, \$9)$, $d(1, \$5)$, $e(2, \$5)$, $f(10, \$10)$, $g(3, \$9)$. For example, item a weighs 3 lbs and is worth a total of \$12. Assume you have a knapsack that can hold a total of 11 lbs. What is an optimal solution to the **fractional** knapsack problem for S ? Show your work.

6. What is the best way to multiply a chain of matrices with dimensions that are 10×5 , 5×2 , 2×20 , 20×12 , 12×4 , and 4×60 ? Show your work (including two tables - one indicating the index k which gives the final multiply index for each subproblem, and the second table which indicates the optimal number of multiplications for each subproblem).

7. Suppose we have a set of activities to schedule among a large number of lecture halls, where any activity can take place in any lecture hall. We wish to schedule all the activities using as few lecture halls as possible. Give an efficient greedy algorithm to determine which activity should use which lecture hall. (This problem is also known as the **interval-graph coloring problem**. We can create an interval graph whose vertices are the given activities and whose edges connect incompatible activities. The smallest number of colors required to color every vertex so that no two adjacent vertices have the same color corresponds to finding the fewest lecture halls needed to schedule all of the given activities.)

8. There are n trading posts along a river, numbered 1 to n as you travel downstream. At any trading post i you can rent a canoe to be returned at any of the downstream trading posts j , where $j \geq i$. You are given a table $R[i, j]$ defining the cost of a canoe which is picked up at post i and dropped off at post j for $1 \leq i \leq j \leq n$. Assume that $R[i, i] = 0$ and that you can't take a canoe upriver (so perhaps $R[i, j] = \infty$ when $i > j$). However, it can happen that the cost of renting from i to j is higher than the total cost of a series of shorter rentals. In this case, you can return the first canoe at some post k between i and j and continue your journey in a second (and maybe third, fourth ...) canoe. There is no extra charge for changing canoes this way. Describe an efficient algorithm to determine the minimum cost of a trip by canoe from each possible departure point i to each possible arrival point j . Analyze the running time of your algorithm.
9. Consider the activity selection problem. Suppose that instead of always selecting the first activity to finish, that we instead selected the activity with the latest start time which is compatible with all previous selected activities. Describe how this approach is a greedy algorithm. Either prove that this approach yields an optimal solution or provide a counterexample to disprove.
10. Consider the activity selection problem. Suppose that instead of always selecting the first activity to finish, that we instead select the activity with the smallest interval (finish time minus start time) which is compatible with all previous selected activities. Either prove that this approach yields an optimal solution or provide a counterexample to disprove.